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Yoshiki Uchida

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Based on Phys. Rev. D 106 (2022) 11, 115033 [2210.08696]

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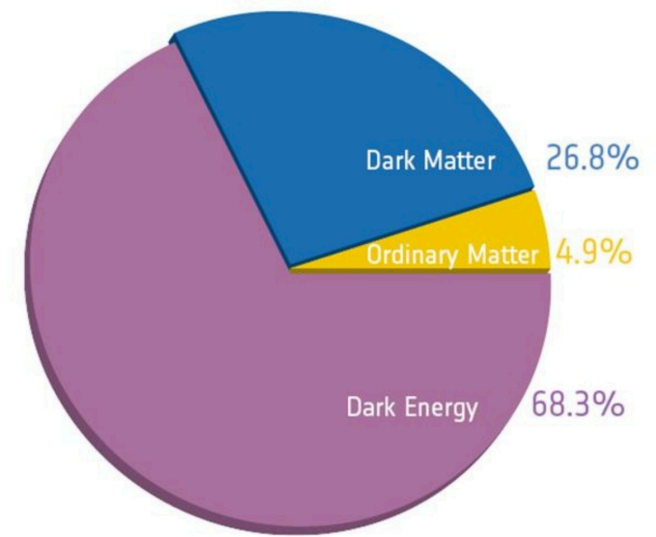
C.W. Chang, K. Tsumura, **YU**, N Yamatsu,  
“Pseudo-Nambu-Goldstone Dark Matter in SU(7) Grand Unification” [2311.13753]

“Kashiwa Dark Matter Symposium 2023” @ Univ. of Tokyo, Kashiwanoha Campus [hybrid]  
2023/12/05-12/08

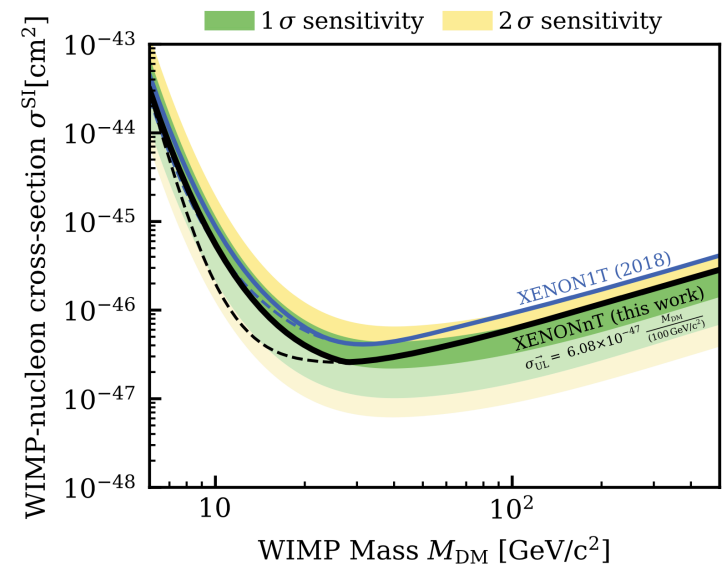
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- An attractive candidate for a DM is ...

**WIMP** ( **W**eakly **I**nteracting **M**assive **P**article )

- thermally produced in the early universe
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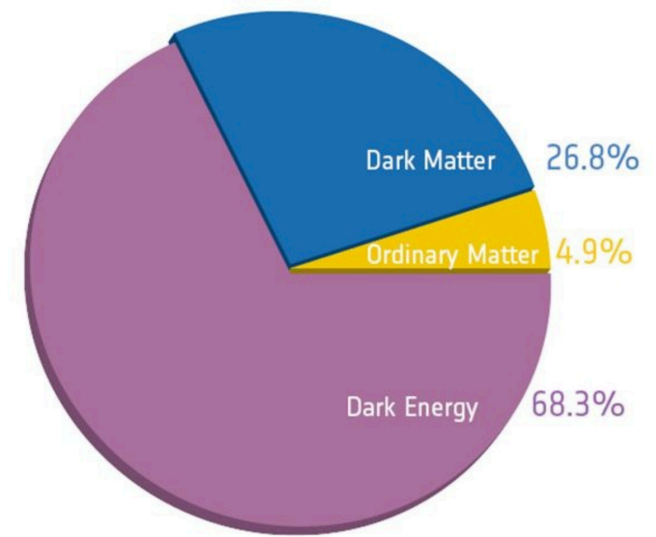
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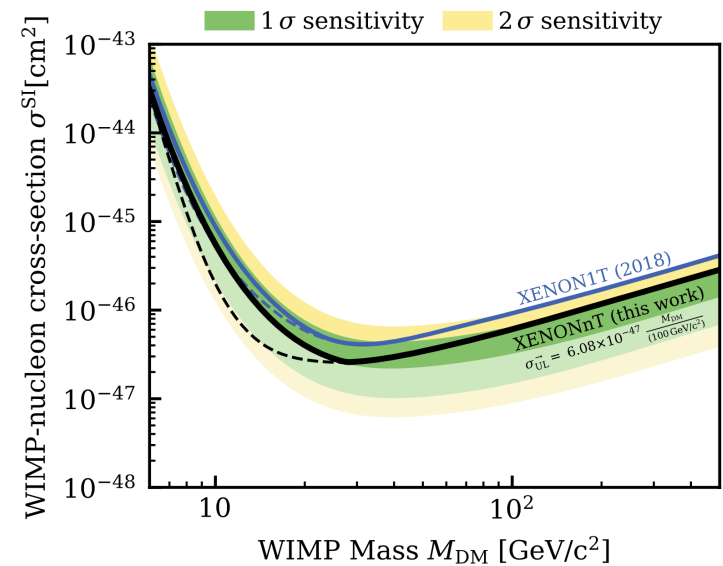
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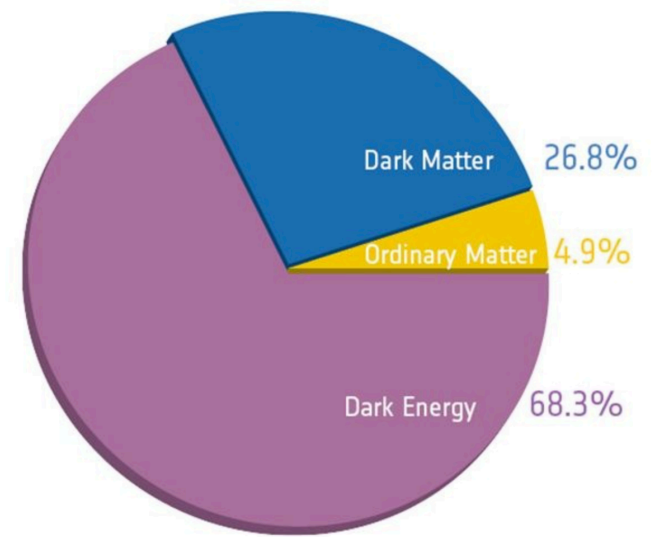


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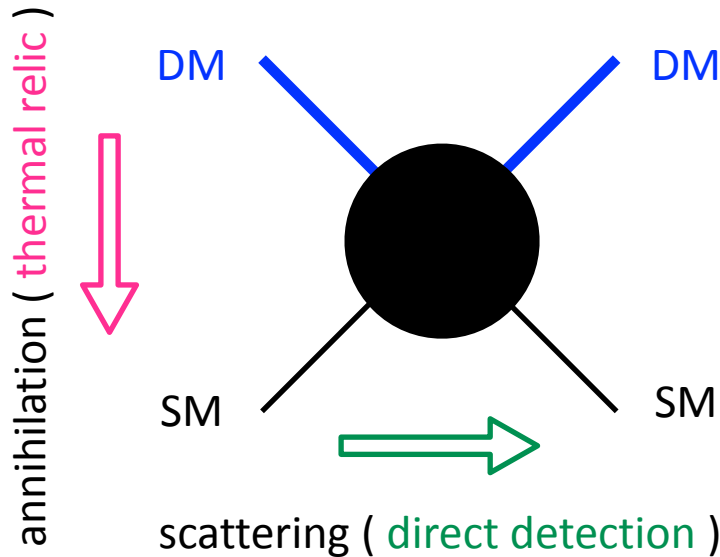
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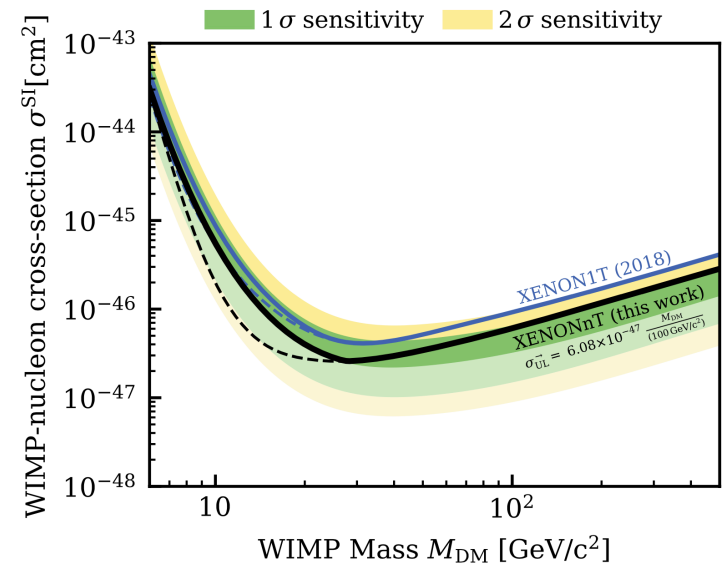
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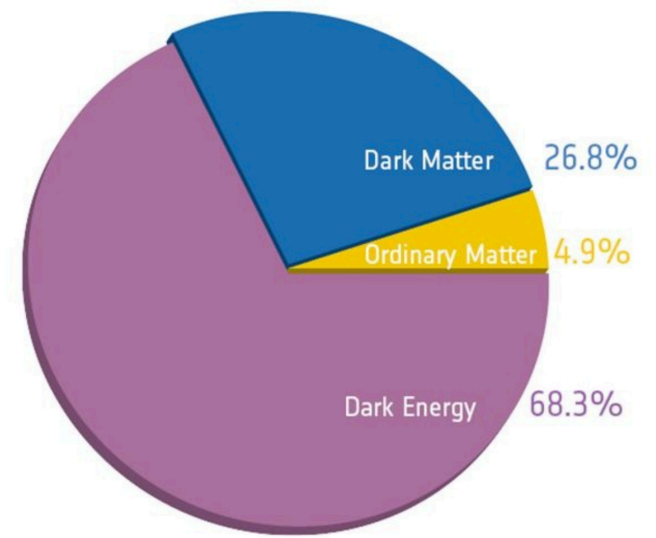


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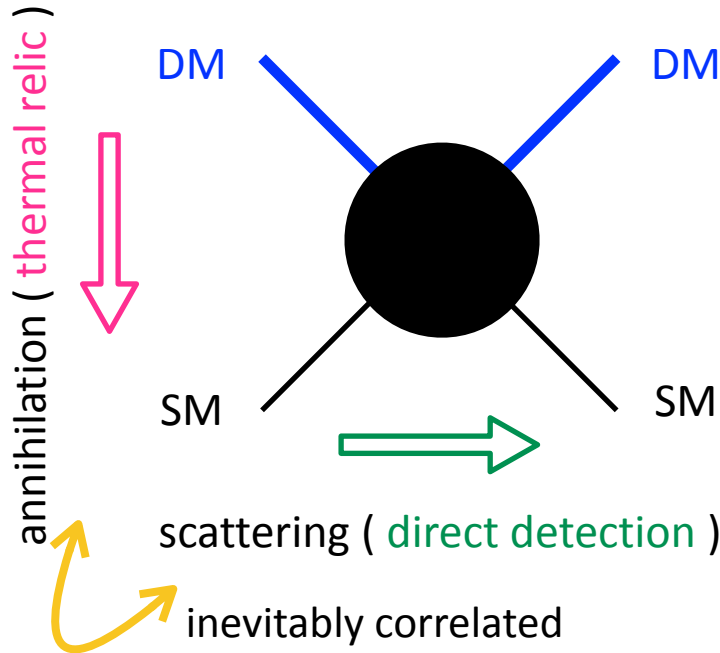
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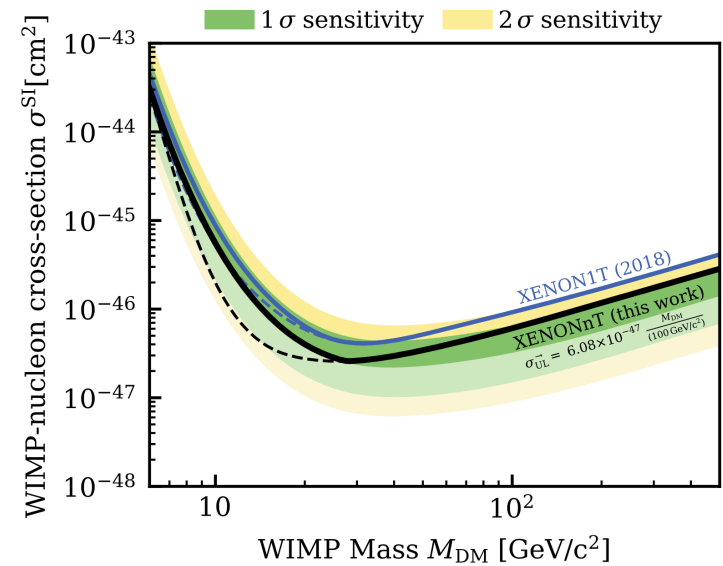
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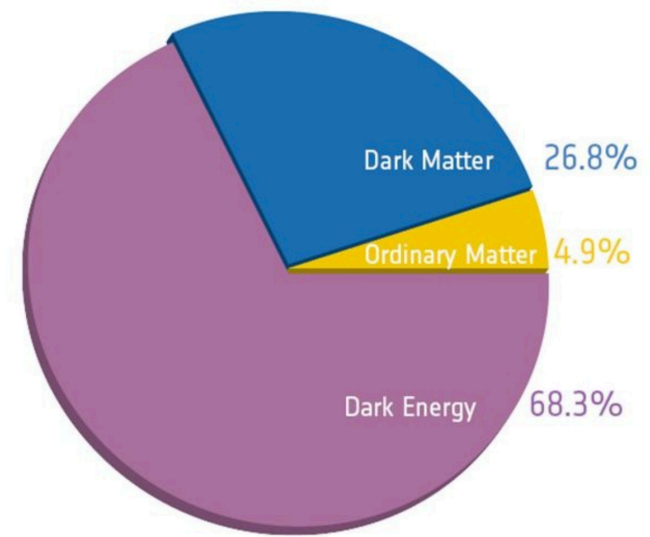


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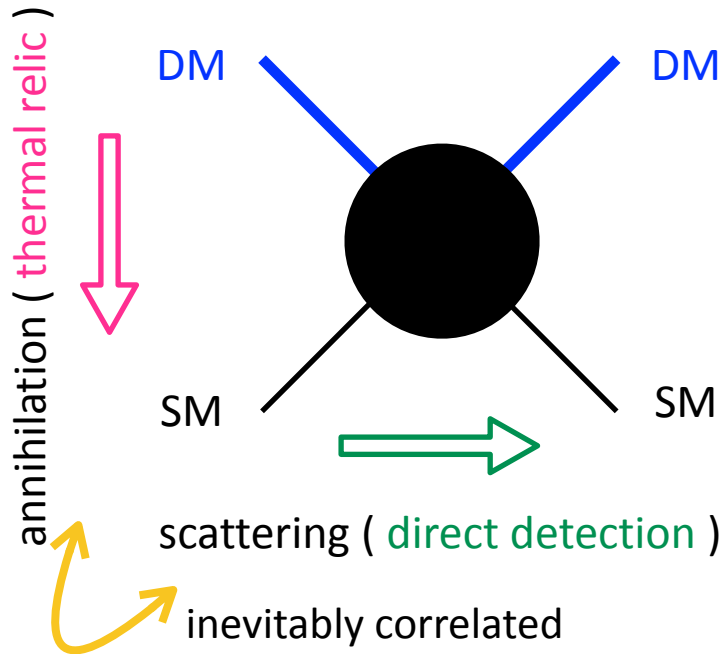
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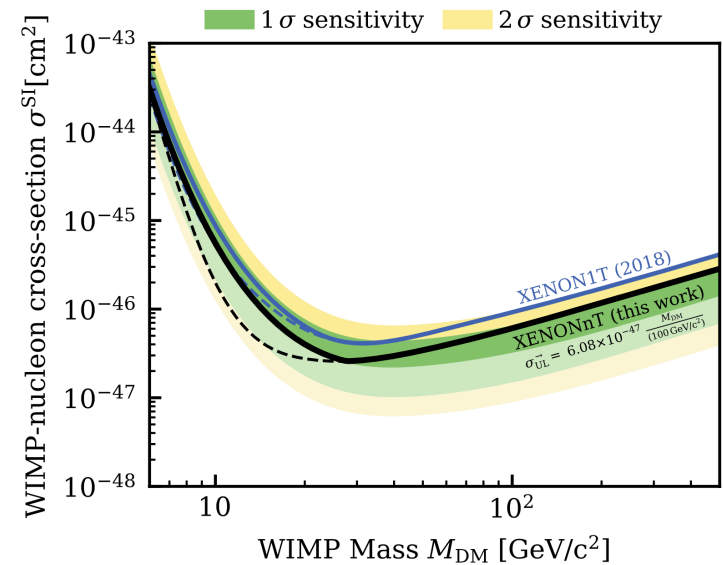
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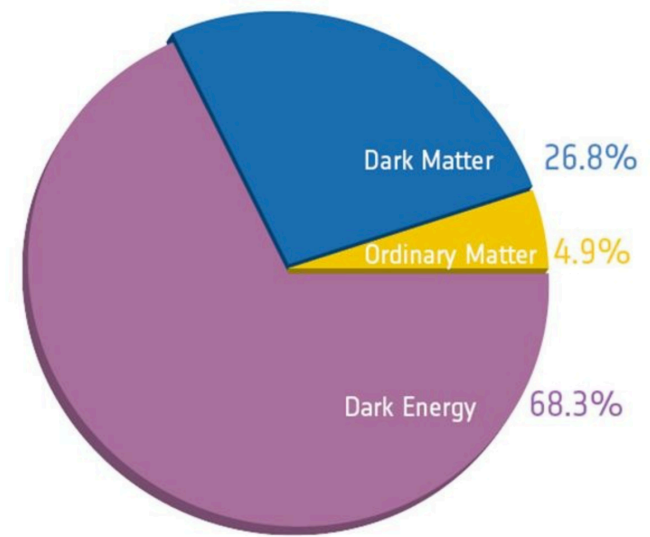
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Stronger DM-SM interaction helps DM to stay longer in thermal bath, leading to  $\Omega h^2 \simeq 0.12$ , but also increases DM-nucleon scattering.

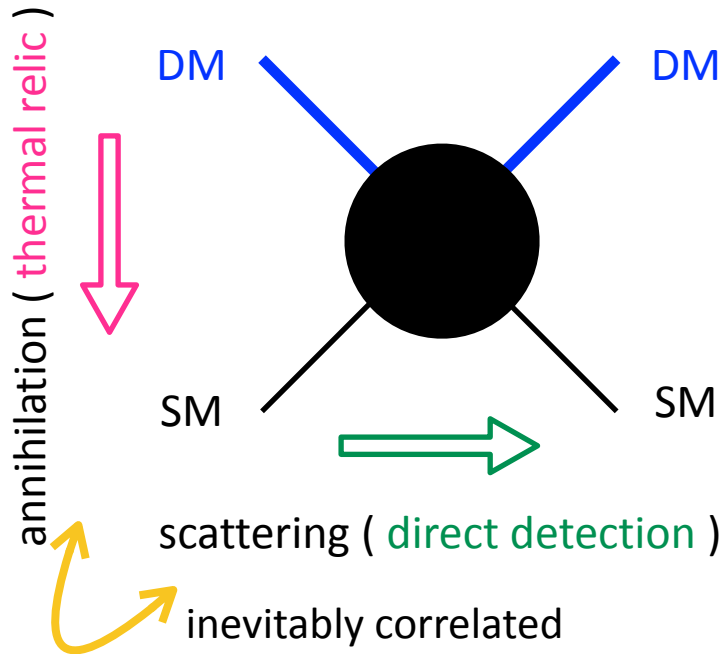
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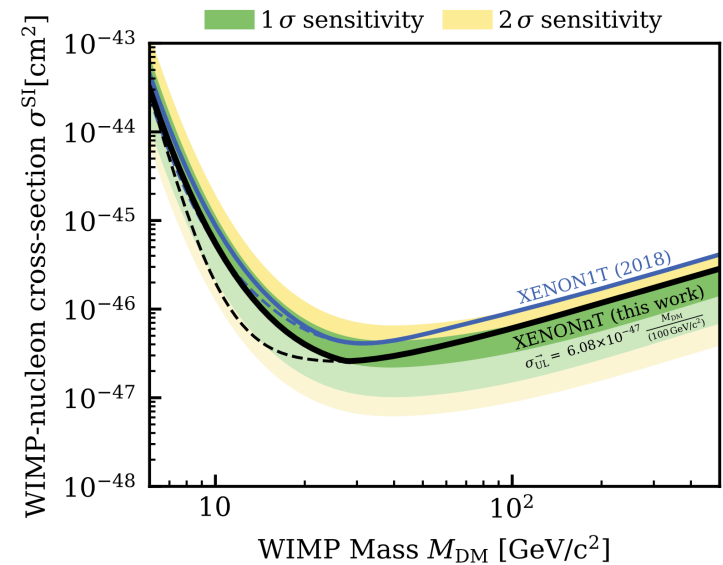
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To realize viable WIMP model, we must address this dilemma.



What type of WIMP model would solve this dilemma?

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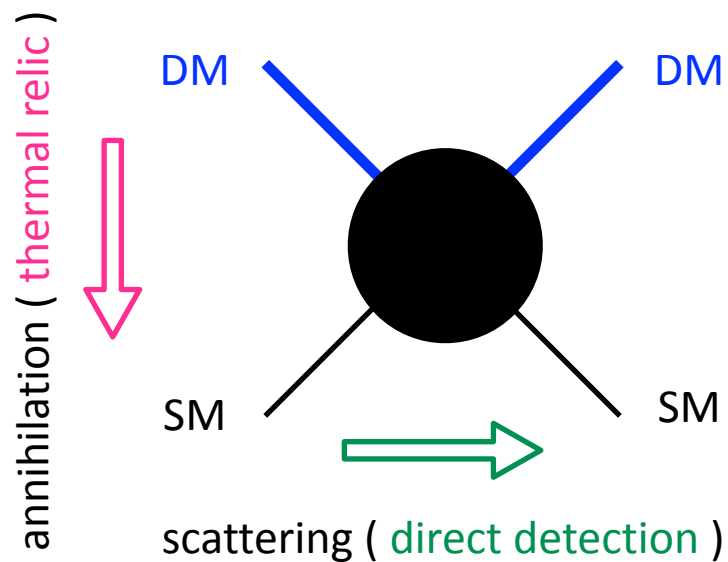
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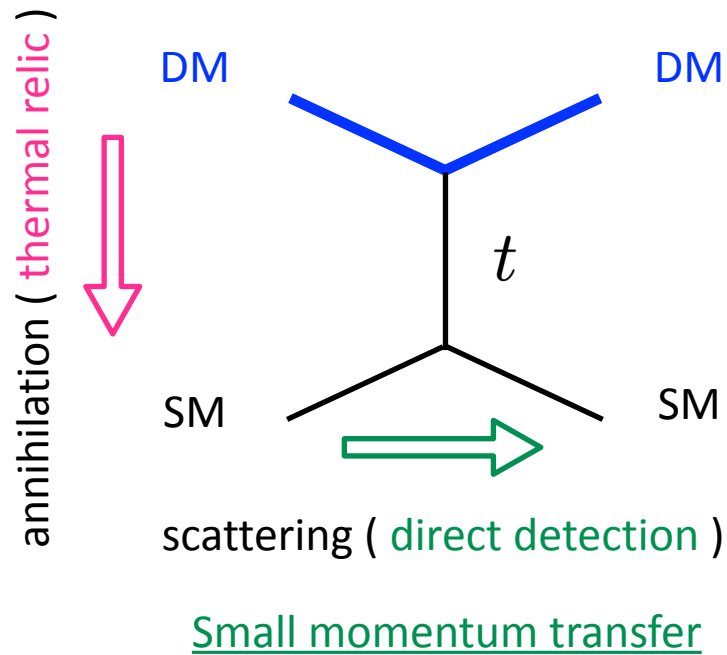
DM communicates with SM particles via derivative interaction



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## pseudo Nambu-Goldstone Boson Dark Matter (pNGB-DM)

symmetry :  $G_{\text{SM}} \times U(1)_{\text{global}}$

new fields : complex  $S \in \mathbf{1}_0$


$$V(H, S) = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \lambda_{HS}|H|^2|S|^2 + \frac{\lambda_S}{2}|S|^4 - \frac{\mu_S'^2}{4}S^2 + \text{h.c.}$$

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Origin for  
pNGB mass   $-\frac{\mu_S^2}{4}S^2 + \text{h.c.}$

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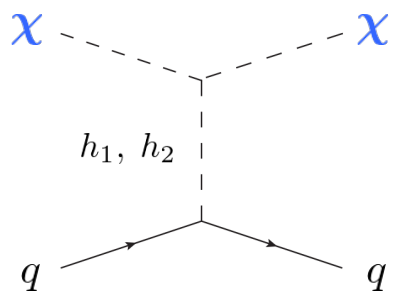
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$\chi$  : pNGB DM

DM-quark scattering



$$\propto \sin\theta \cos\theta \left( \frac{m_{h_2}^2}{t - m_{h_2}^2} - \frac{m_{h_1}^2}{t - m_{h_1}^2} \right) \xrightarrow{t \rightarrow 0} 0$$

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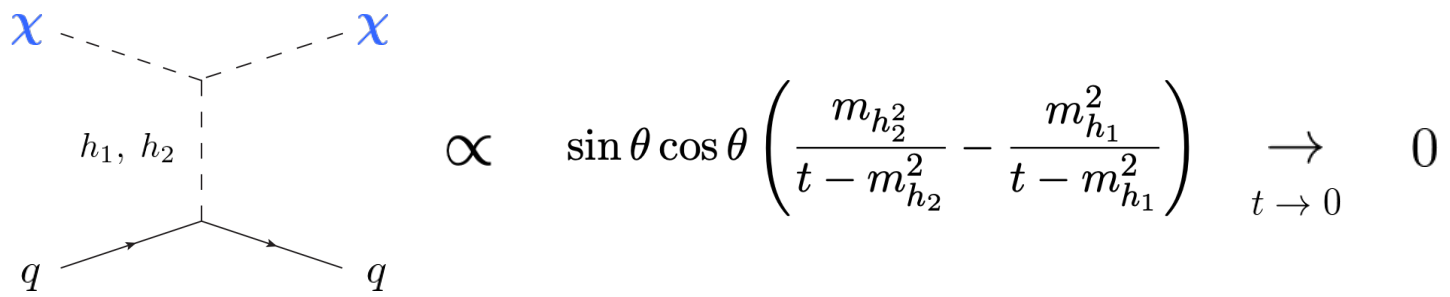
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Original pNGB DM model has several problems to be solved ...



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To construct a feasible model, we need appropriate UV completions

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**Solutions :** gauged  $U(1)_{B-L}$  model

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	$Q_L$	$L$	$u_R^c$	$d_R^c$	$e_R^c$	$\nu_R^c$	$H$	$S_1$	$S_2$
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$U(1)_Y$	+1/6	-1/2	-2/3	+1/3	+1	0	+1/2	0	0
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①  $V(H, S_1, S_2) \ni \kappa S_2^* S_1^2 + \text{h.c.} \rightarrow \frac{1}{2} m_{DM}^2 \chi^2$

The other soft-breaking term are forbidden by  $U(1)_{B-L}$

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The other soft-breaking term are forbidden by  $U(1)_{B-L}$

②  $Z_2$  is embedded in  $U(1)_{B-L}$  gauge symmetry

**pseudo Nambu-Goldstone Boson Dark Matter (pNGB-DM)**

$$V(H, S) = V_{SM}(H) + \lambda_{HS}|H|^2|S|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_S}{2}|S|^4 - \left( \frac{\mu_S'}{4}S^2 + \text{h.c.} \right)$$

- Problems :**
- ①  $\mu_S''S^3, \mu_S'''|S|^2S, \dots$  are dropped by hands ✓
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- Solutions :** gauged  $U(1)_{B-L}$  model
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	$Q_L$	$L$	$u_R^c$	$d_R^c$	$e_R^c$	$\nu_R^c$	$H$	$S_1$	$S_2$
$SU(2)_L$	<b>2</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>1</b>
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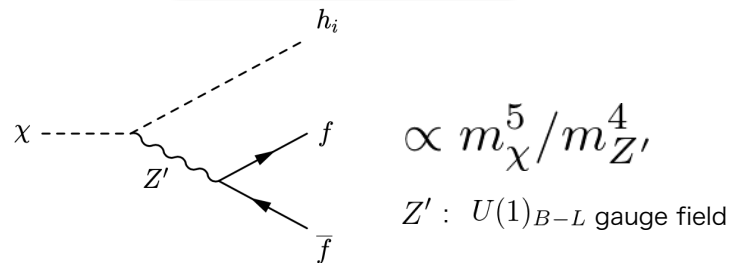
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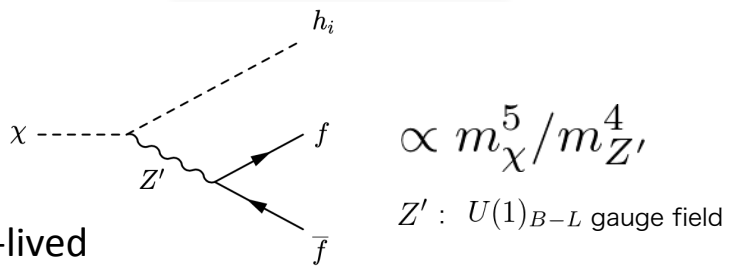
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$v_{B-L} \simeq 10^{15} \text{ GeV}$   
 "hierarchy problem"

$V(H, S_1, S_2) \ni \kappa S_2^* S_1^2 + \text{h.c.}$

➔ **pNGB DM decays**



Higher  $U(1)_{B-L}$  breaking scale required to make DM long-lived

# Brief Summary

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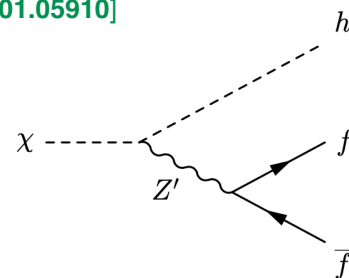
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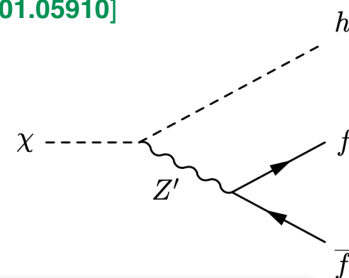
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Goal

UV completion of pNGB-DM model with no large hierarchy

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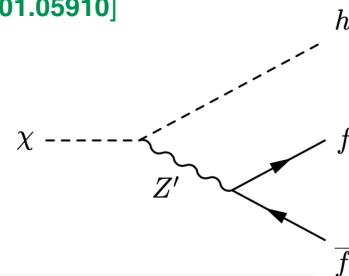
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UV completion of pNGB-DM model with no large hierarchy

We want to explain the origin of pNGB mass,  
and want a stable DM so that we don't need to introduce large hierarchy

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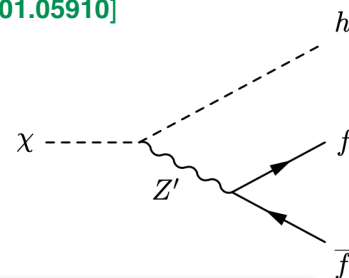
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UV completion of pNGB-DM model with no large hierarchy

**Hint** : SM scalar sector  $G_{\text{SM}} = SU(2)_L \times U(1)_Y$

$$V_{\text{SM}}(H) = -\mu_H^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

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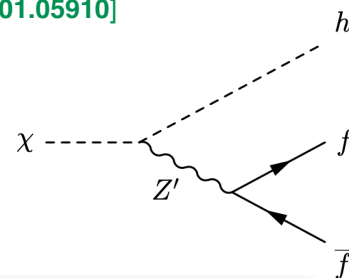
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$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

accidental global symmetry :  $G_{\text{global}} = O(4) \simeq SU(2)_L \times SU(2)_R$

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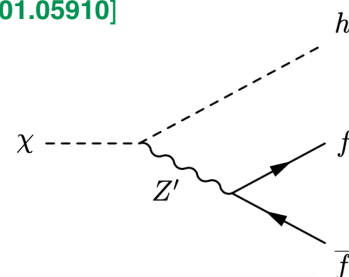
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accidental global symmetry :  $G_{\text{global}} = O(4) \simeq SU(2)_L \times SU(2)_R \xrightarrow{\langle H \rangle \neq 0} SU(2)_V$  : custodial sym.

# Our Model

Goal

UV completion of pNGB-DM model with no large hierarchy

H. Otsuka, K. Tsumura, [YU](#), N. Yamatsu, Phys. Rev. D 106 (2022) 11, 115033 [[2210.08696](#)]

We consider  $G_{\text{SM}} \times SU(2)_D^{\text{gauge}}$  symmetry  
 and introduce  $\Phi \in \mathbf{2}$ ,  $\Delta \in \mathbf{3}$  under  $SU(2)_D^{\text{gauge}}$   
 $\Sigma = (\tilde{\Phi}, \Phi)$

	$SU(2)_L$	$U(1)_Y$	$SU(2)_D^{\text{gauge}}$
$H$	$\mathbf{2}$	$1/2$	$\mathbf{1}$
$\Phi$	$\mathbf{1}$	$0$	$\mathbf{2}$
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$$V(H, \Phi, \Delta) \quad \Sigma = (\tilde{\Phi}, \Phi)$$

$$= -\mu_H^2 H^\dagger H - \frac{1}{2}\mu_\Phi^2 \text{Tr} [\Sigma^\dagger \Sigma] - \frac{1}{2}\mu_\Delta^2 \text{Tr} [\Delta^2]$$

Mass terms

$$+ \lambda_H (H^\dagger H)^2 + \frac{\lambda_\Phi}{4} (\text{Tr} [\Sigma^\dagger \Sigma])^2 + \frac{\lambda_\Delta}{4} (\text{Tr} [\Delta^2])^2$$

4-point self-int.

$$+ \lambda_{H\Phi} (H^\dagger H) \text{Tr} [\Sigma^\dagger \Sigma] + \lambda_{H\Delta} (H^\dagger H) \text{Tr} [\Delta^2] + \frac{\lambda_{\Phi\Delta}}{2} \text{Tr} [\Sigma^\dagger \Sigma] \text{Tr} [\Delta^2]$$

4-point int.

$$- \sqrt{2}\kappa \text{Tr} [\sigma_3 \Sigma^\dagger \Delta \Sigma]$$

# $V(H, \Phi, \Delta)$ has “Dark custodial symmetry”

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$\Delta$	$\mathbf{1}$	$0$	$\mathbf{3}$

$$V(H, \Phi, \Delta) \quad \Sigma = (\tilde{\Phi}, \Phi)$$

$$= -\mu_H^2 H^\dagger H - \frac{1}{2}\mu_\Phi^2 \text{Tr} [\Sigma^\dagger \Sigma] - \frac{1}{2}\mu_\Delta^2 \text{Tr} [\Delta^2]$$

Mass terms

$$+ \lambda_H (H^\dagger H)^2 + \frac{\lambda_\Phi}{4} (\text{Tr} [\Sigma^\dagger \Sigma])^2 + \frac{\lambda_\Delta}{4} (\text{Tr} [\Delta^2])^2$$

4-point self-int.

$$+ \lambda_{H\Phi} (H^\dagger H) \text{Tr} [\Sigma^\dagger \Sigma] + \lambda_{H\Delta} (H^\dagger H) \text{Tr} [\Delta^2] + \frac{\lambda_{\Phi\Delta}}{2} \text{Tr} [\Sigma^\dagger \Sigma] \text{Tr} [\Delta^2]$$

4-point int.

$$- \sqrt{2}\kappa \text{Tr} [\sigma_3 \Sigma^\dagger \Delta \Sigma]$$

# $V(H, \Phi, \Delta)$ has “Dark custodial symmetry”

H. Otsuka, K. Tsumura, [YU](#), N. Yamatsu, Phys. Rev. D 106 (2022) 11, 115033 [[2210.08696](#)]

We consider  $G_{\text{SM}} \times SU(2)_D^{\text{gauge}}$  symmetry  
 and introduce  $\Phi \in \mathbf{2}$ ,  $\Delta \in \mathbf{3}$  under  $SU(2)_D^{\text{gauge}}$

	$SU(2)_L$	$U(1)_Y$	$SU(2)_D^{\text{gauge}}$
$H$	<b>2</b>	1/2	<b>1</b>
$\Phi$	<b>1</b>	0	<b>2</b>
$\Delta$	<b>1</b>	0	<b>3</b>

$$V(H, \Phi, \Delta) \quad \Sigma = (\tilde{\Phi}, \Phi)$$

$$\begin{aligned}
 &= -\mu_H^2 H^\dagger H - \frac{1}{2}\mu_\Phi^2 \text{Tr} [\Sigma^\dagger \Sigma] - \frac{1}{2}\mu_\Delta^2 \text{Tr} [\Delta^2] \\
 &+ \lambda_H (H^\dagger H)^2 + \frac{\lambda_\Phi}{4} (\text{Tr} [\Sigma^\dagger \Sigma])^2 + \frac{\lambda_\Delta}{4} (\text{Tr} [\Delta^2])^2 \\
 &+ \lambda_{H\Phi} (H^\dagger H) \text{Tr} [\Sigma^\dagger \Sigma] + \lambda_{H\Delta} (H^\dagger H) \text{Tr} [\Delta^2] + \frac{\lambda_{\Phi\Delta}}{2} \text{Tr} [\Sigma^\dagger \Sigma] \text{Tr} [\Delta^2] \\
 &- \sqrt{2}\kappa \text{Tr} [\sigma_3 \Sigma^\dagger \Delta \Sigma]
 \end{aligned}$$

Invariant under  
 global “Dark custodial symmetry”

$$\begin{aligned}
 \Delta &\rightarrow U_L^{\text{dark}} \Delta U_L^{\text{dark} \dagger} \\
 \Sigma &\rightarrow U_L^{\text{dark}} \Sigma U_R^{\text{dark} \dagger}
 \end{aligned}
 \quad (H \rightarrow H)$$

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$\Delta$	<b>1</b>	0	<b>3</b>

$$V(H, \Phi, \Delta) \quad \Sigma = (\tilde{\Phi}, \Phi)$$

$$= -\mu_H^2 H^\dagger H - \frac{1}{2}\mu_\Phi^2 \text{Tr} [\Sigma^\dagger \Sigma] - \frac{1}{2}\mu_\Delta^2 \text{Tr} [\Delta^2]$$

$$+ \lambda_H (H^\dagger H)^2 + \frac{\lambda_\Phi}{4} (\text{Tr} [\Sigma^\dagger \Sigma])^2 + \frac{\lambda_\Delta}{4} (\text{Tr} [\Delta^2])^2$$

$$+ \lambda_{H\Phi} (H^\dagger H) \text{Tr} [\Sigma^\dagger \Sigma] + \lambda_{H\Delta} (H^\dagger H) \text{Tr} [\Delta^2] + \frac{\lambda_{\Phi\Delta}}{2} \text{Tr} [\Sigma^\dagger \Sigma] \text{Tr} [\Delta^2]$$

$$- \sqrt{2}\kappa \text{Tr} [\sigma_3 \Sigma^\dagger \Delta \Sigma]$$

Explicitly breaks

global “Dark custodial symmetry”

$$\Delta \rightarrow U_L^{\text{dark}} \Delta U_L^{\text{dark}\dagger} \quad (H \rightarrow H)$$

$$\Sigma \rightarrow U_L^{\text{dark}} \Sigma U_R^{\text{dark}\dagger}$$

# $V(H, \Phi, \Delta)$ has “Dark custodial symmetry”

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$$V(H, \Phi, \Delta) \quad \Sigma = (\tilde{\Phi}, \Phi)$$

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 &= -\mu_H^2 H^\dagger H - \frac{1}{2}\mu_\Phi^2 \text{Tr} [\Sigma^\dagger \Sigma] - \frac{1}{2}\mu_\Delta^2 \text{Tr} [\Delta^2] \\
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 &+ \lambda_{H\Phi} (H^\dagger H) \text{Tr} [\Sigma^\dagger \Sigma] + \lambda_{H\Delta} (H^\dagger H) \text{Tr} [\Delta^2] + \frac{\lambda_{\Phi\Delta}}{2} \text{Tr} [\Sigma^\dagger \Sigma] \text{Tr} [\Delta^2]
 \end{aligned}$$

$$- \sqrt{2}\kappa \text{Tr} [\sigma_3 \Sigma^\dagger \Delta \Sigma]$$

Explicitly breaks

global “Dark custodial symmetry”

Even after  $\langle \Phi \rangle \neq 0$  &  $\langle \Delta \rangle \neq 0$ , the exact  $U(1)_{\text{global}}$  remains unbroken

$$V(H, \Phi, \Delta) = -\mu_H^2 H^\dagger H - \frac{1}{2}\mu_\Phi^2 \text{Tr} [\Sigma^\dagger \Sigma] - \frac{1}{2}\mu_\Delta^2 \text{Tr} [\Delta^2]$$

$$+ \lambda_H (H^\dagger H)^2 + \frac{\lambda_\Phi}{4} (\text{Tr} [\Sigma^\dagger \Sigma])^2 + \frac{\lambda_\Delta}{4} (\text{Tr} [\Delta^2])^2$$

$$+ \lambda_{H\Phi} (H^\dagger H) \text{Tr} [\Sigma^\dagger \Sigma] + \lambda_{H\Delta} (H^\dagger H) \text{Tr} [\Delta^2] + \frac{\lambda_{\Phi\Delta}}{2} \text{Tr} [\Sigma^\dagger \Sigma] \text{Tr} [\Delta^2]$$

$$- \sqrt{2}\kappa \text{Tr} [\sigma_3 \Sigma^\dagger \Delta \Sigma]$$

$$\Sigma = (\tilde{\Phi}, \Phi)$$

H. Otsuka, K. Tsumura, [YU](#), N. Yamatsu,  
Phys. Rev. D 106 (2022) 11, 115033 [[2210.08696](#)]

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$\Delta$	<b>1</b>	0	<b>3</b>

Global symmetry breaking pattern

$\Phi \in \mathbf{2}$ ,  $\Delta \in \mathbf{3}$  under  $SU(2)_D^{\text{gauge}}$

- **Approximate** symmetry

$$SU(2)_L^{\text{dark}} \times SU(2)_R^{\text{dark}} \xrightarrow{\langle \Phi \rangle \neq 0} SU(2)_V^{\text{dark}} \xrightarrow{\langle \Delta \rangle \neq 0} U(1)_V^{\text{dark}}$$

$$\# \text{ of broken generators} = (3+3) - 1 = 5$$

Total NGBs

- **Exact** symmetry

$$SU(2)_L^{\text{dark}} \times U(1)_R^{\text{dark}} \xrightarrow{\langle \Phi \rangle \neq 0} U(1)_V^{\text{dark}}$$

$$\# \text{ of broken generators} = (3+1) - 1 = 3$$

would-be NGBs

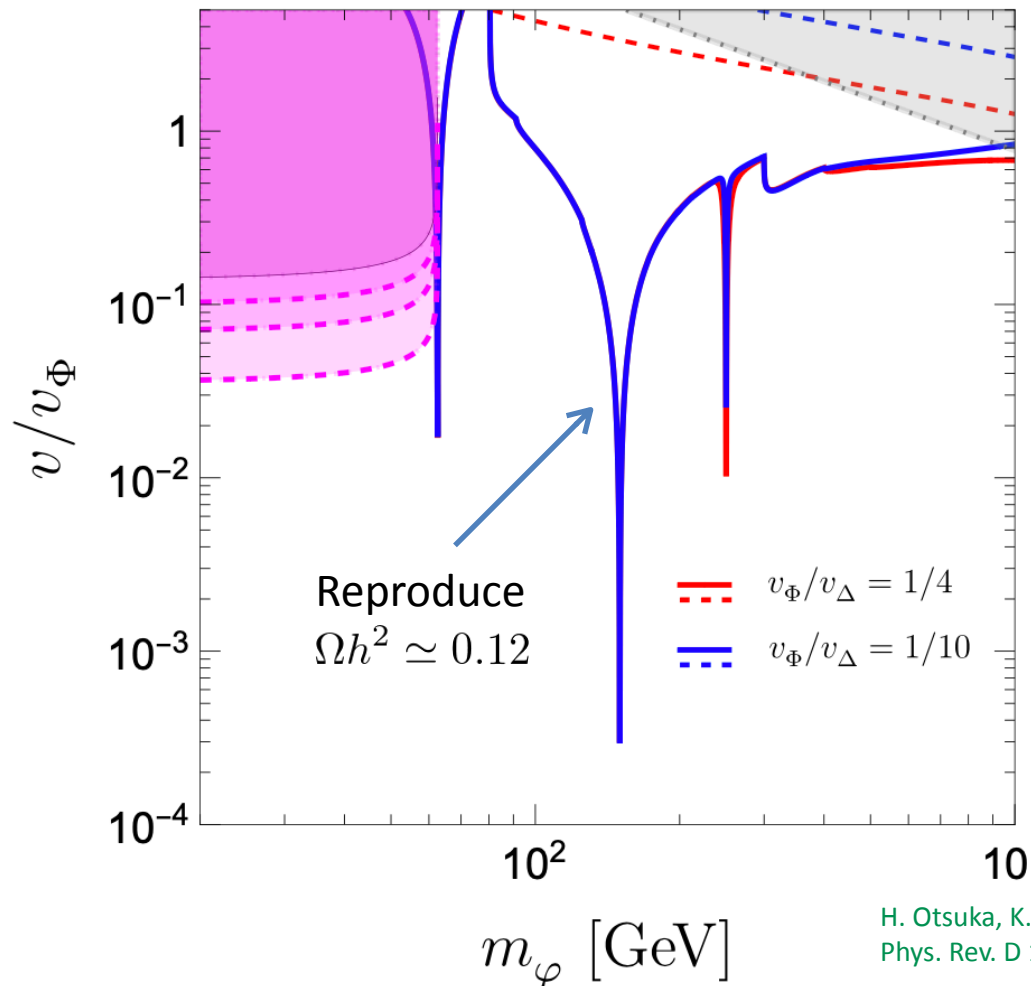
# of pNGB is **2** (= 5 - 3)  $\Rightarrow$  complex pNGB with  $U(1)_V^{\text{dark}}$  charge

# Benchmark

scalar mass :  $(m_{h_1}, m_{h_2}, m_{h_3}) = (125, 300, 500) \text{ GeV}$

mixing angle :  $(\sin \alpha_x, \sin \alpha_y, \sin \alpha_z) = (0.06, 0.05, 0.1)$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_x & \sin \alpha_x \\ 0 & -\sin \alpha_x & \cos \alpha_x \end{pmatrix} \begin{pmatrix} \cos \alpha_y & 0 & \sin \alpha_y \\ 0 & 1 & 0 \\ -\sin \alpha_y & 0 & \cos \alpha_y \end{pmatrix} \begin{pmatrix} \cos \alpha_z & \sin \alpha_z & 0 \\ -\sin \alpha_z & \cos \alpha_z & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h \\ \phi_3 \\ \eta_3 \end{pmatrix}$$

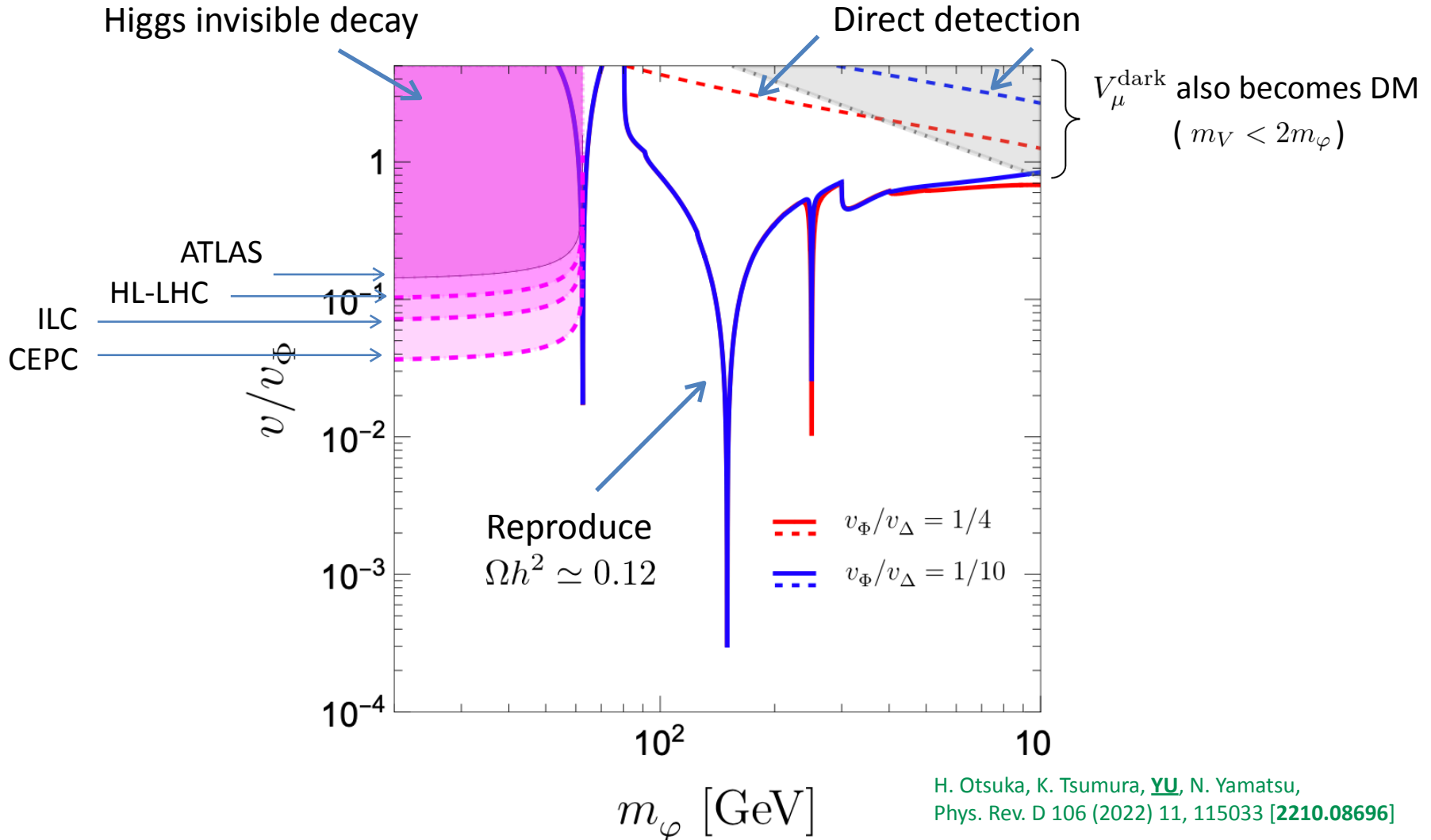


Benchmark

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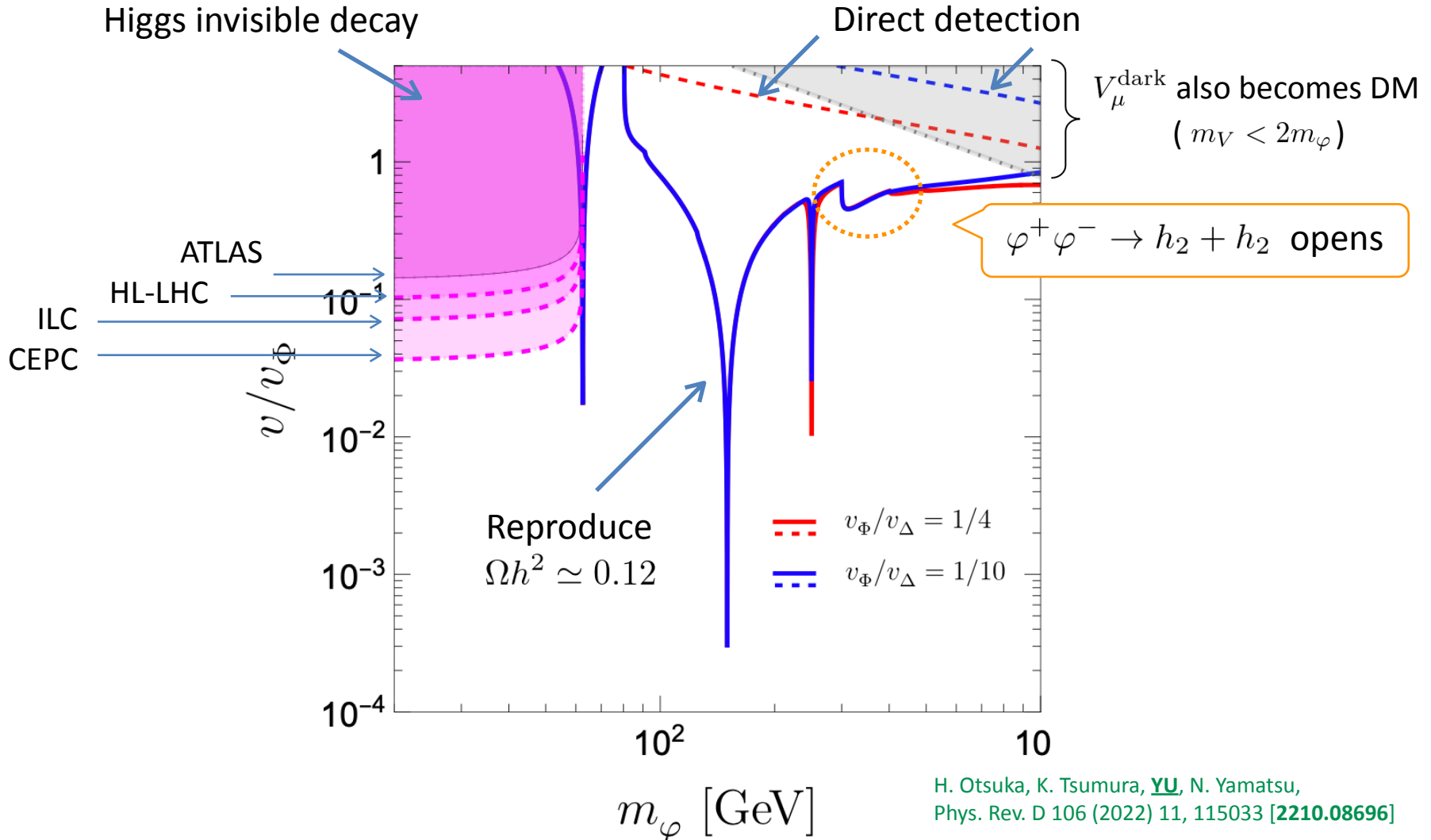


Benchmark

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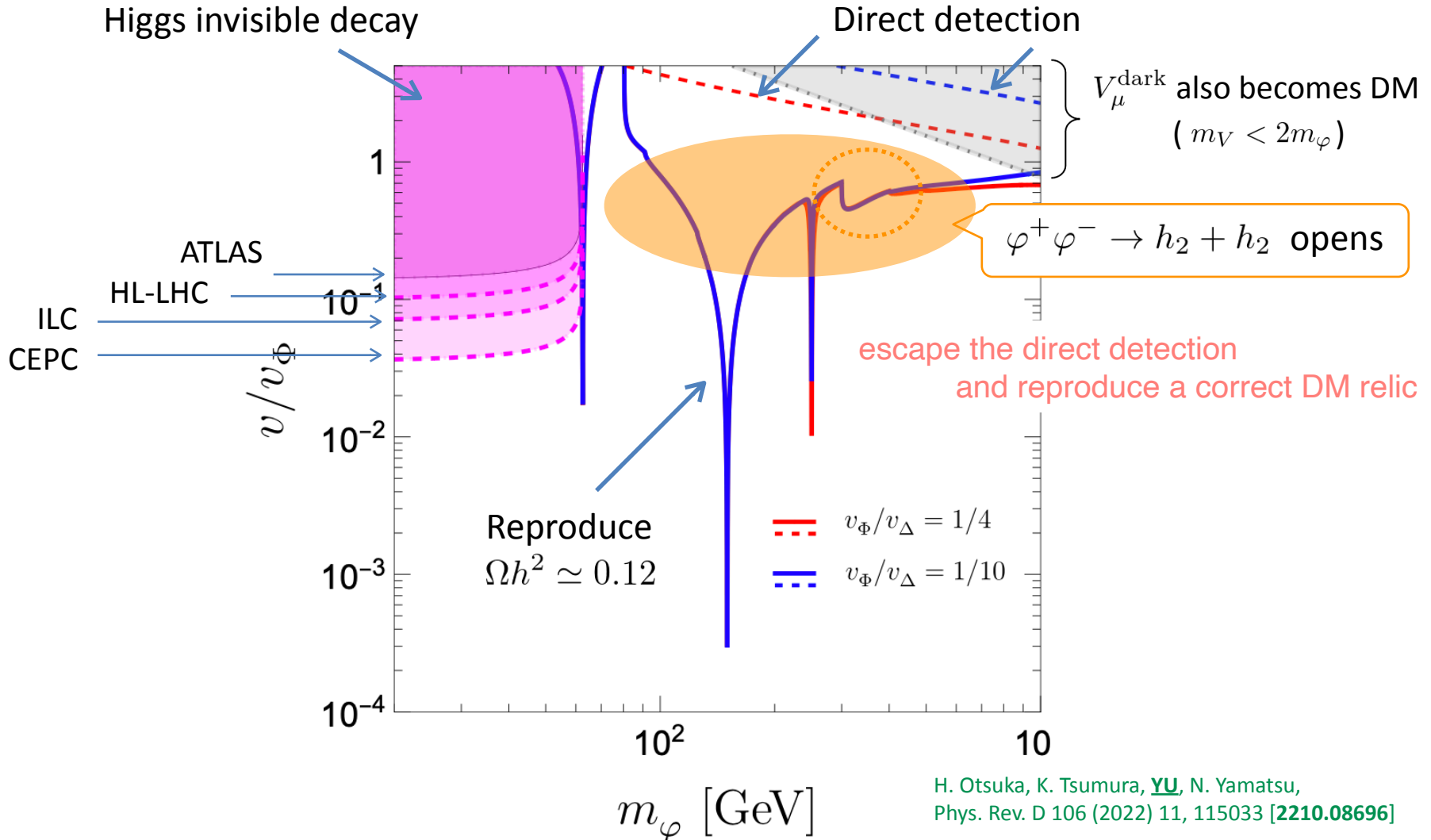


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# Summary

- In the original abelian pNGB DM model, particular soft-breaking terms are included, and their origins are not addressed.
- UV completed models are proposed, but all of them predict decaying DM. In order to make DM long-lived, we must introduce large hierarchy in symmetry breaking scales.
- We construct pNGB-DM model with non-abelian gauge symmetry. Unbroken dark custodial symmetry ensure stability of pNGB DM. We don't need to introduce large hierarchy.

# Back Up

But, this is not the end of the story ...

Three-point breaking term may spoil the cancellation

- **Two-point** breaking term T. Abe and Y. Hamada, [2205.11919]

$\mu_\chi^2 (\phi^\dagger T^3 \phi) \Rightarrow$  Origin of pNGBs' mass

$\propto t = (p_2 - p_1)^2 \xrightarrow{t \rightarrow 0} 0$

- **Three-point** breaking term Our Model

$\kappa \Phi^\dagger \Delta \Phi \quad \begin{matrix} \Phi \in 2 \\ \Delta \in 3 \end{matrix} \Rightarrow$  Origin of pNGBs' mass & **interactions**

$\propto t \text{ (pentagon)} + \kappa \text{ (diamond)}$

We must make sure *DM-nucleon scattering is suppressed enough*

# Soft breaking terms

- Soft-breaking = Quadratic

$$V_{\text{SM}}(H) + \lambda_{HS}|H|^2|S|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_S}{2}|S|^4 - \left( \frac{\mu_S'^2}{4}S^2 + \text{h.c.} \right)$$

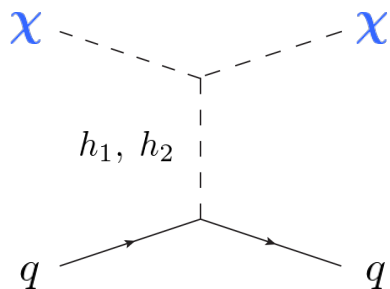
C. Gross, O. Lebedev, and T. Toma, Phys. Rev. Lett. 119 (2017) 19, 191801, [1708.02253]

- Soft-breaking = Quadratic + tadpole

$$V_{\text{SM}}(H) + \lambda_{HS}|H|^2|S|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_S}{2}|S|^4 - \left( \frac{\mu_S'^2}{4}S^2 + \text{h.c.} \right) + (aS + \text{h.c.})$$

V. Barger, P. Langacker, M. McCaskey, M. Ramsey-Musolf and G. Shaughnessy, Phys. Rev. D 79 (2009), 015018, [0811.0393]

G. C. Cho, C. Idegawa and E. Senaha, Phys. Lett. B 823 (2021), 136787, [2105.11830]



$$\propto \left\{ \left( -\frac{m_{h_1}^2}{t - m_{h_1}^2} + \frac{m_{h_2}^2}{t - m_{h_2}^2} \right) \simeq 0 \quad @ \quad t \rightarrow 0 \right. \\ \left. + \frac{\sqrt{2}a}{v_S} \left( -\frac{1}{t - m_{h_1}^2} + \frac{1}{t - m_{h_2}^2} \right) \right\} \simeq 0 \quad @ \quad m_{h_1} \simeq m_{h_2}$$

Based on Idegawa-san's slide