The classical field approximation for Ultra Light Dark Matter

arxiv.2310.07119

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Kashiwa DM 2023
Dec, 5th 2023

Ultra light dark matter

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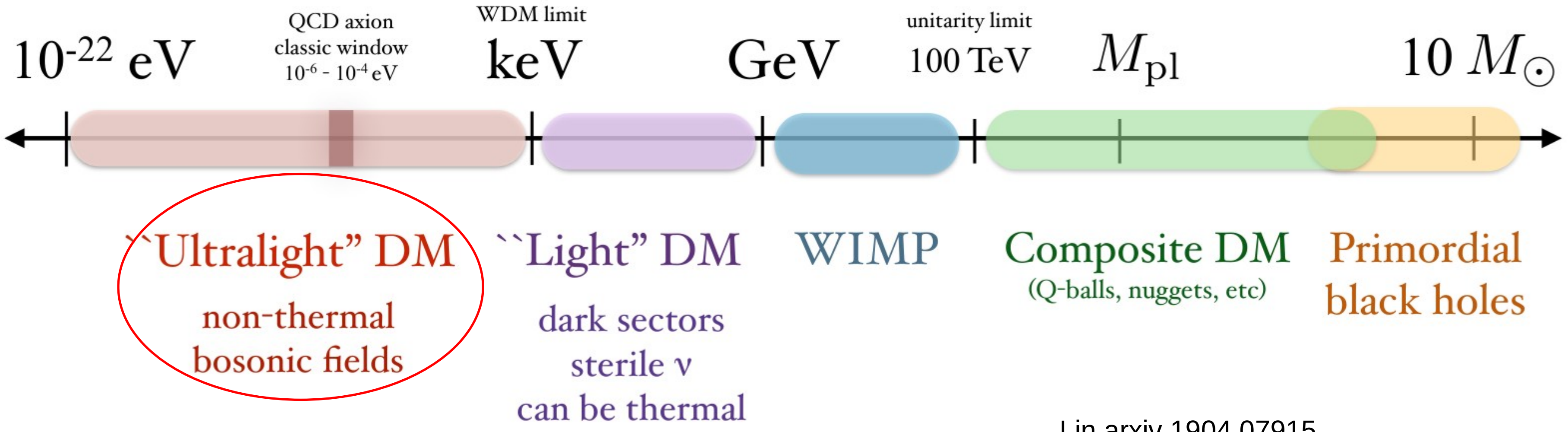


Lin arxiv 1904.07915

Ultra light dark matter

- At the lowest masses dark matter manifests wave-like phenomena on astrophysical scales

$$m \lesssim 10^{-19}\,\mathrm{eV}$$

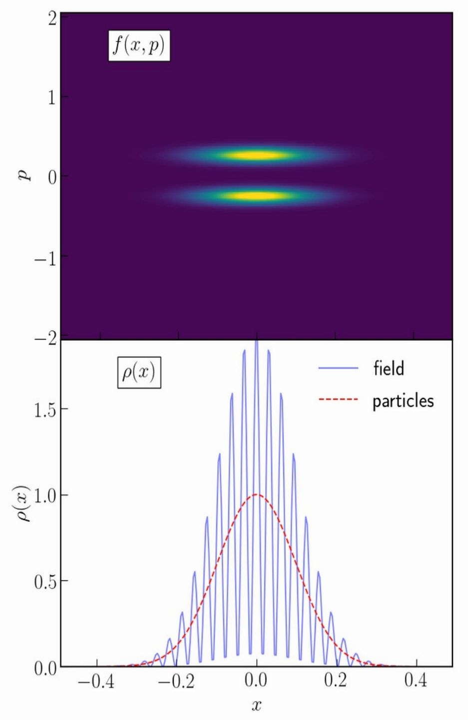
$$\lambda = \frac{2\pi\hbar}{mv} = 0.48\,\mathrm{kpc}\left(\frac{10^{-22}\,\mathrm{eV}}{m}\right)\left(\frac{250\,\mathrm{km/s}}{v}\right)$$

Ultra light dark matter

- At the lowest masses dark matter manifests wave-like phenomena on astrophysical scales
  - Interference patterns

$$\lambda_d \sim \frac{\hbar}{\Delta p}$$

Density fields for different particle masses



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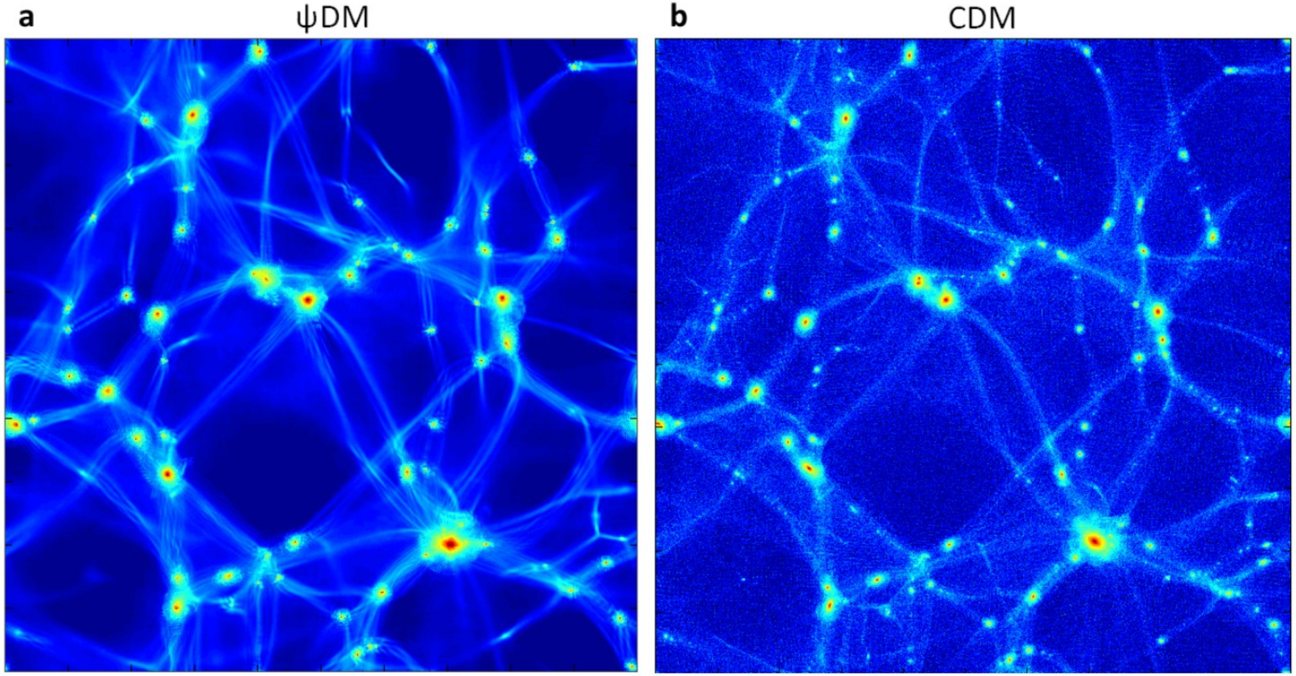
Schive et al (Nature 2014)



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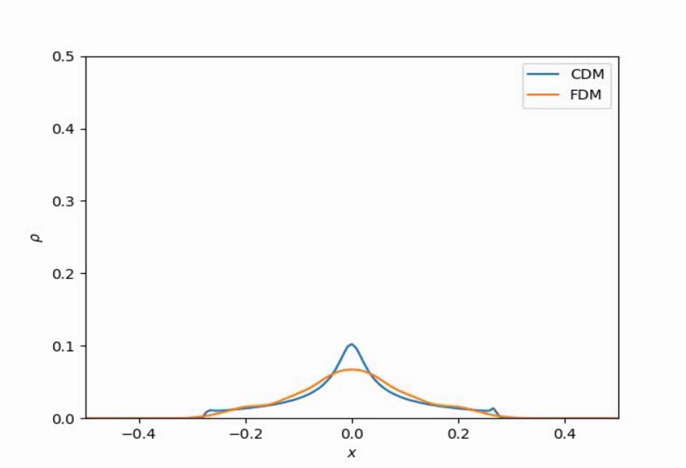
  - “Quantum” pressure

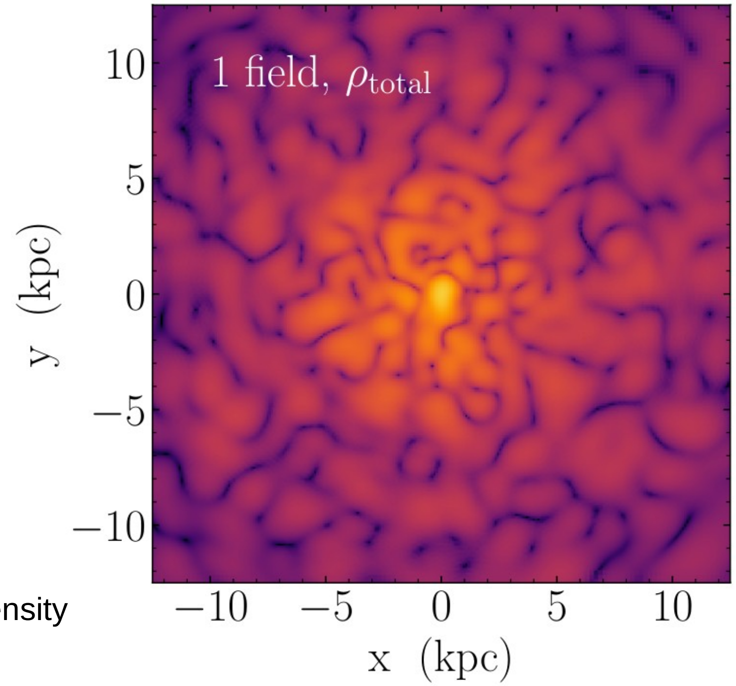


Gravitational collapse in 1D

Ultra light dark matter

- At the lowest masses dark matter manifests wave-like phenomena on astrophysical scales
  - Interference patterns
  - “Quantum” pressure
  - Granular density patterns

Typical halo density



Gosenca [,**Eberhardt**] et al., PRD (2023)

Ultra light dark matter

- At the lowest masses dark matter manifests wave-like phenomena on astrophysical scales
- Gives a rich phenomenology of constraints

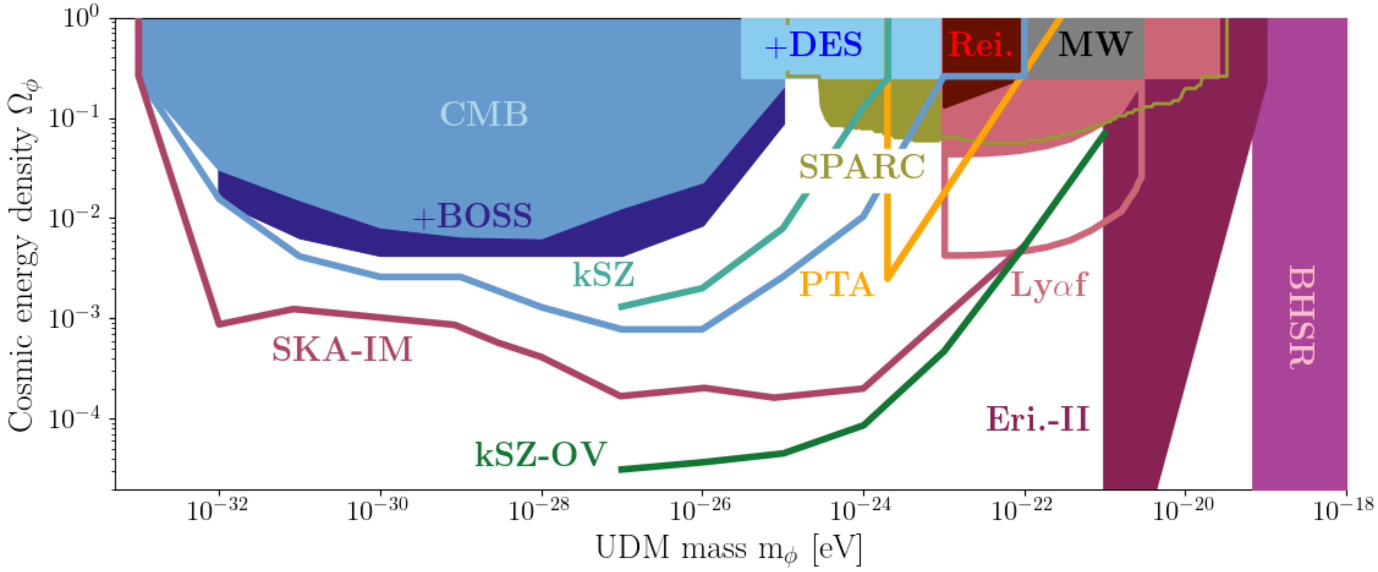


Image credit: Kier Rogers

Ultra light dark matter

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Method
Stellar dispersions
Galaxy density profiles
Satellite abundance/mass

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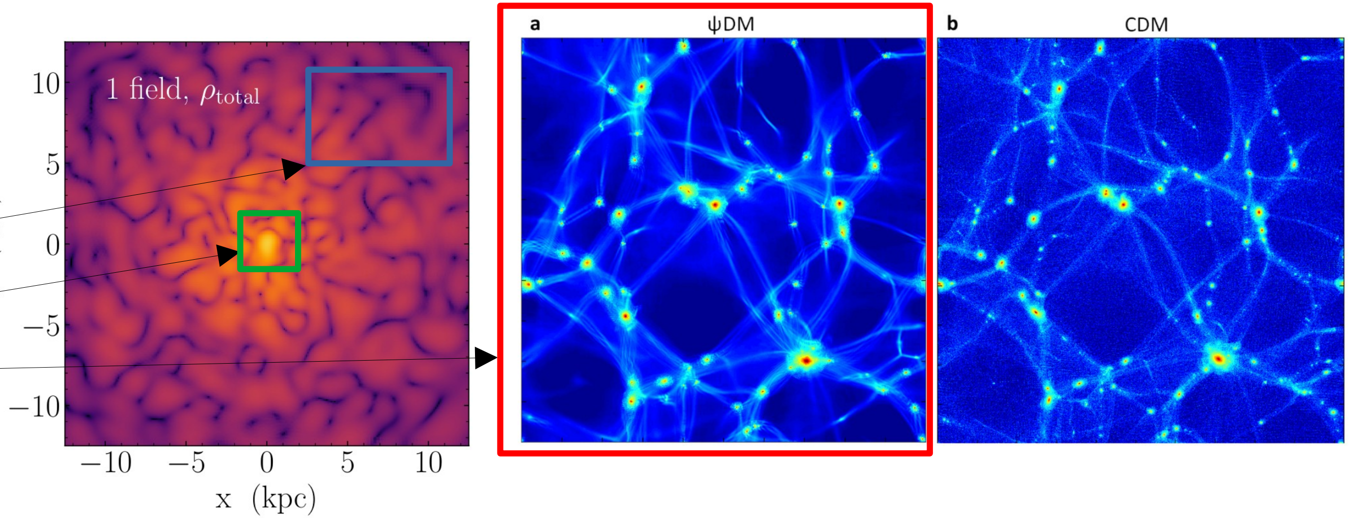

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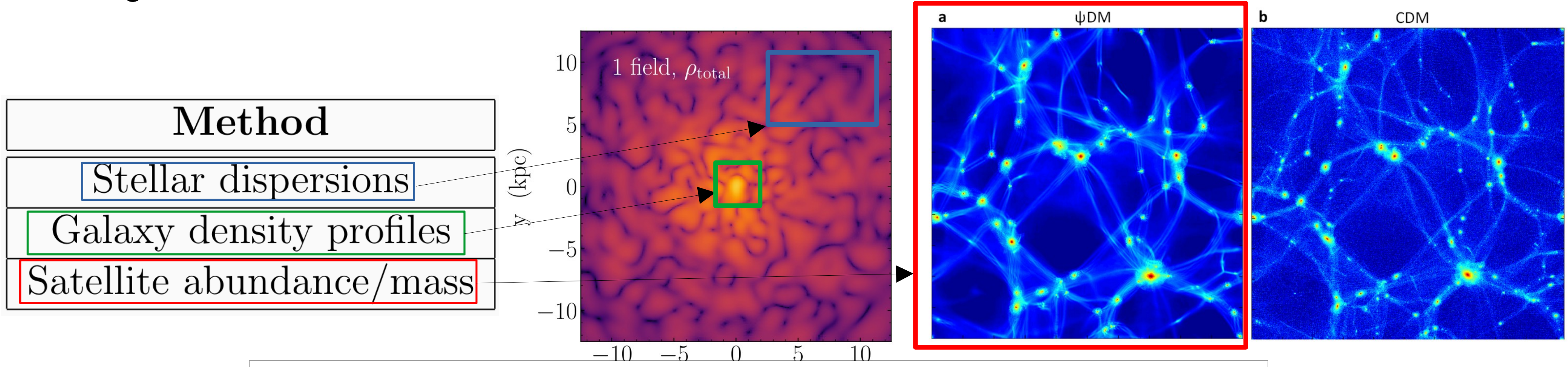

Understanding these simulations is essential to understanding the model and its constraints

Simulations

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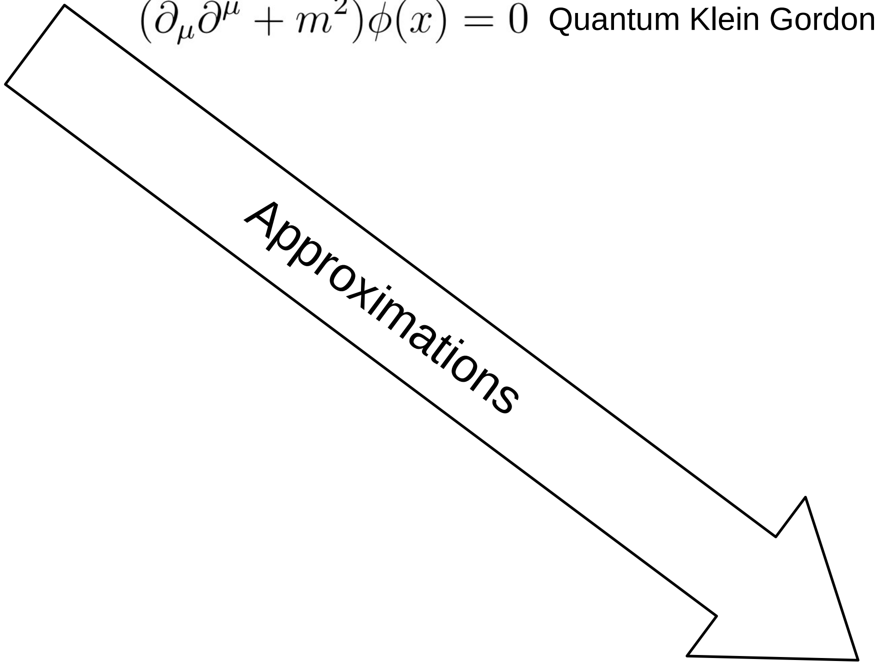

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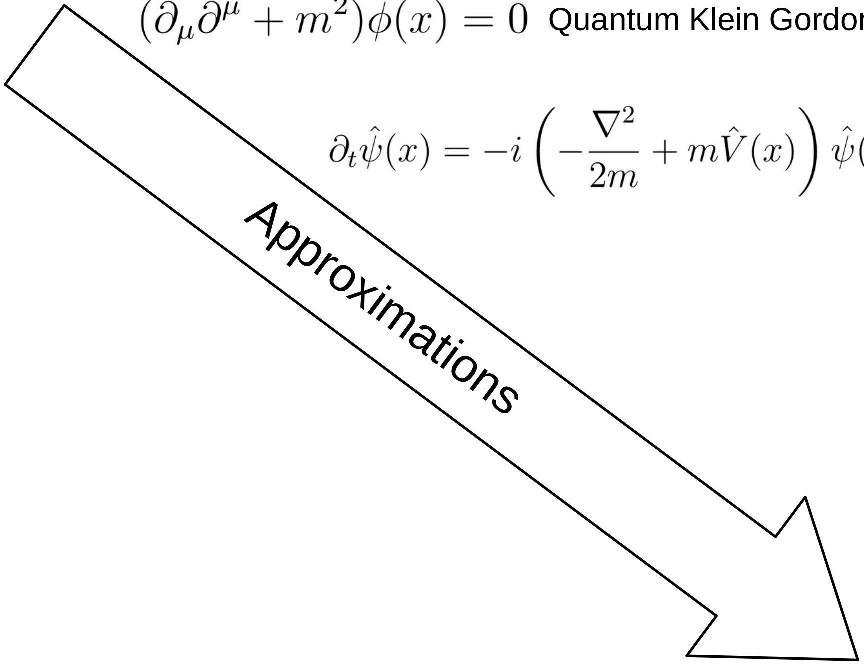
Non relativistic

$$\partial_t \hat{\psi}(x) = -i\left(-\frac{\nabla^2}{2m} + m\hat{V}(x)\right)\hat{\psi}(x)$$ Quantum Schrodinger equation



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$\partial_t \psi(x_g) = -i\left(-\frac{\nabla^2}{2m} + mV(x_g)\right)\psi(x_g)$

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Predictions are made here

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$$\hat{\psi} \to \psi$$

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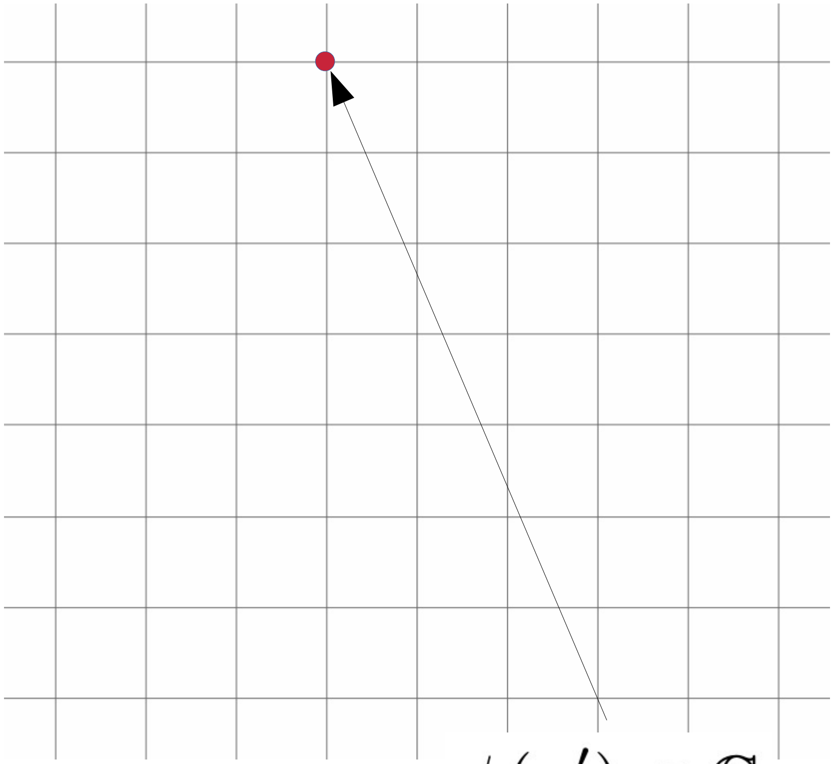
- A classical field places a number at every point in space



$\psi(x') \in \mathbb{C}$

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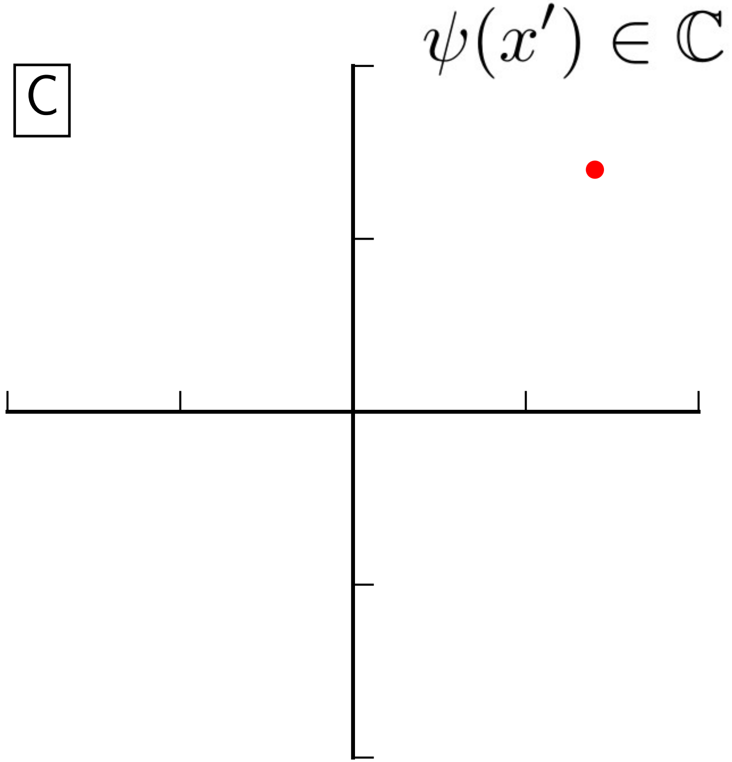

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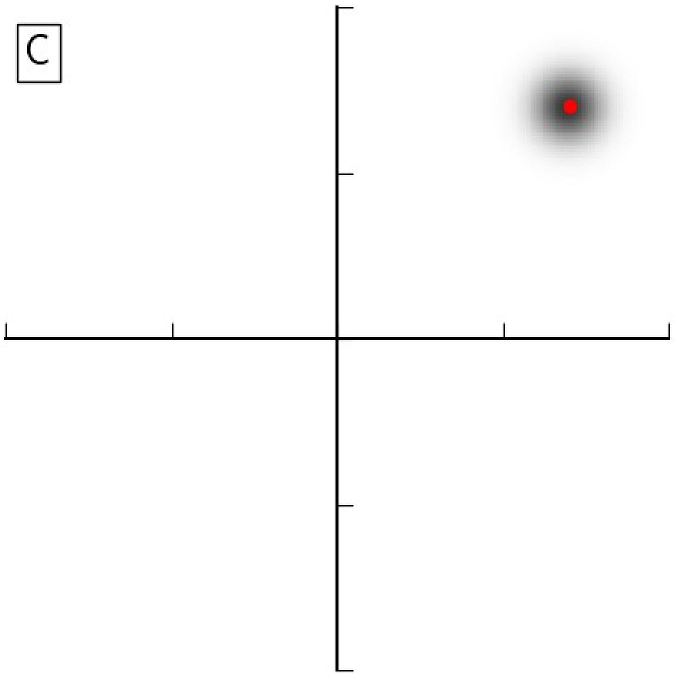
Coherent state



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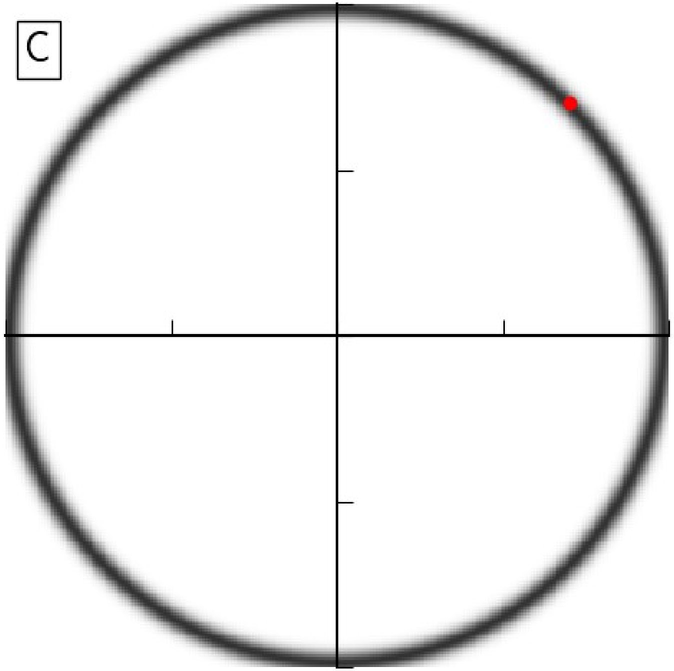
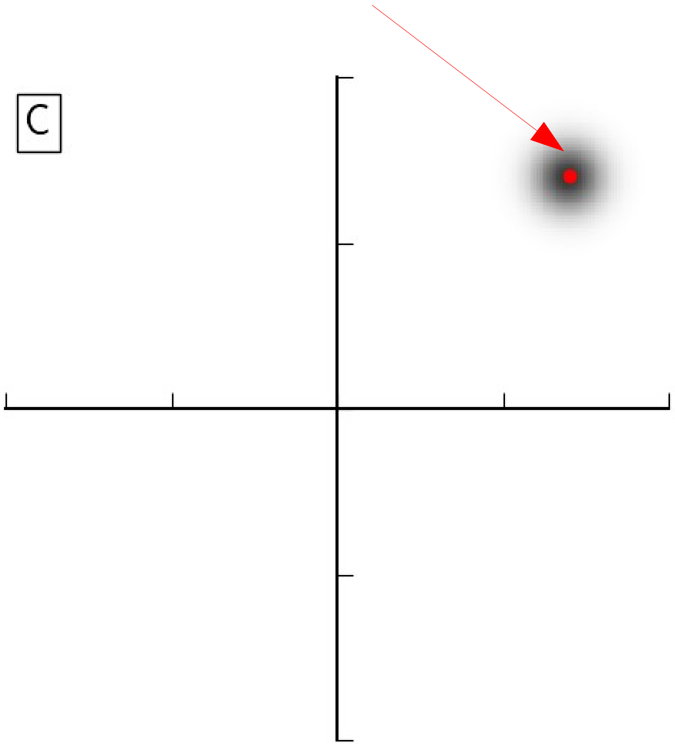
Number eigenstate



Classical approximation

$$\langle\hat{\psi}(x')\rangle = \psi(x')$$



- Classical field theory is an approximation of quantum field theory replacing operators with numbers
- A classical field places a number at every point in space
- The quantum field places a probability distribution at each point
- If the distribution is tightly peaked around the classical value then we can approximate the distribution using this number

Classical approximation

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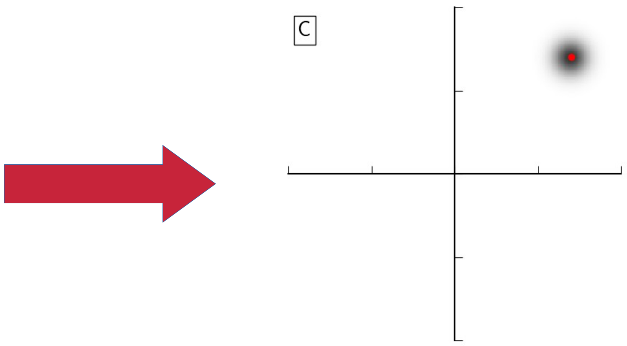
  - The misalignment mechanism produces a quantum coherent state (specifies a distribution shape)

Coherent state



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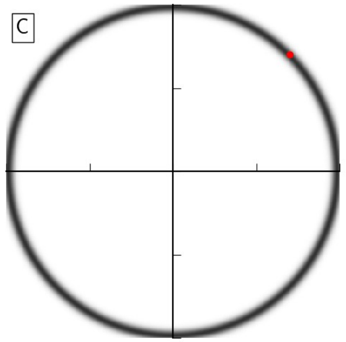

Classical approximation

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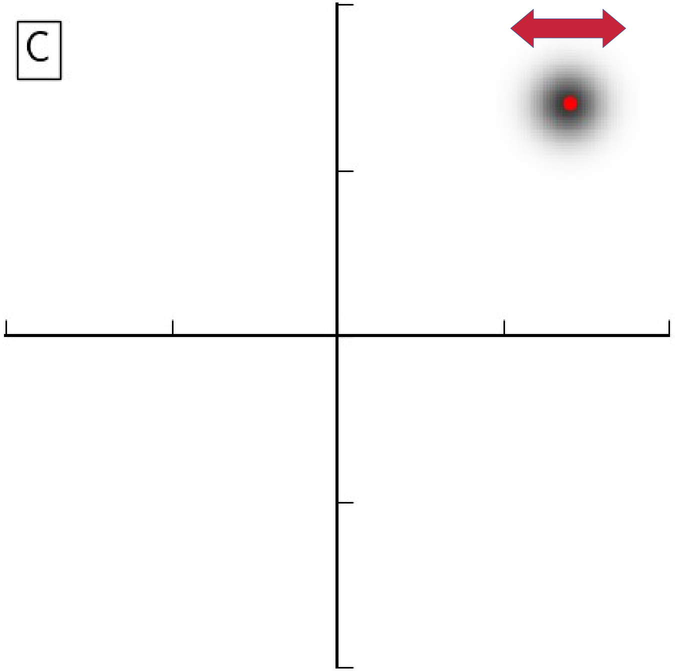
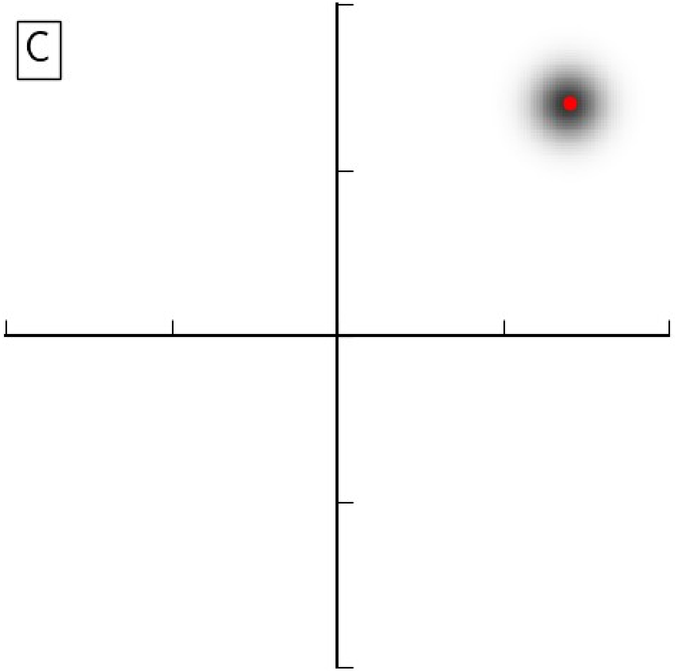
  - The misalignment mechanism produces a quantum coherent state
  - Occupation numbers are very large (gives the fractional variance)



$$\frac{\langle \hat{n} \rangle}{\sqrt{\mathrm{Var}(\hat{n})}} \sim \frac{1}{\sqrt{n_{tot}}} \qquad n_{tot} \sim 10^{100}$$

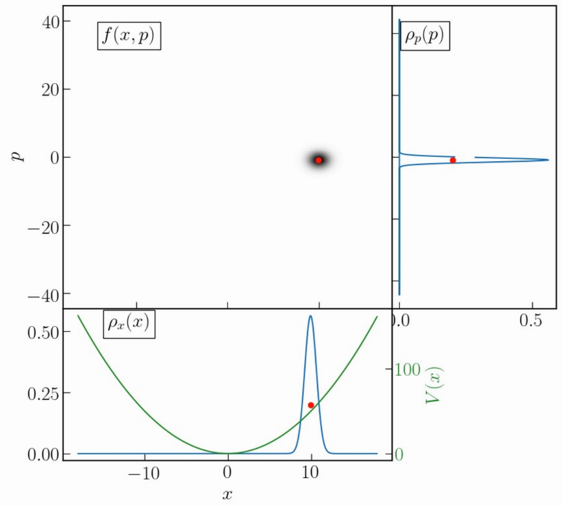
Classical approximation

- The classical field approximation is usually motivated in two ways
  - The misalignment mechanism produces a quantum coherent state
  - Occupation numbers are very large
- Both conditions are necessary for the classical field equations to make accurate predictions [Eberhardt et al (PRD 2021)]



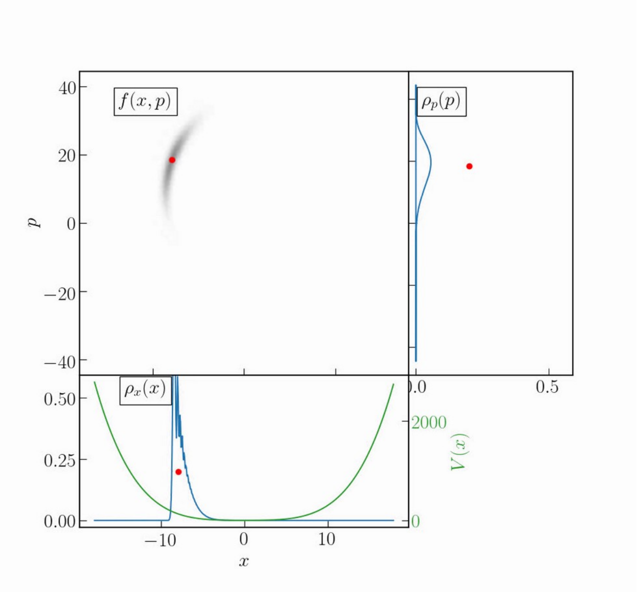
Classical approximation

- In the absence of nonlinearities we would expect this description to survive



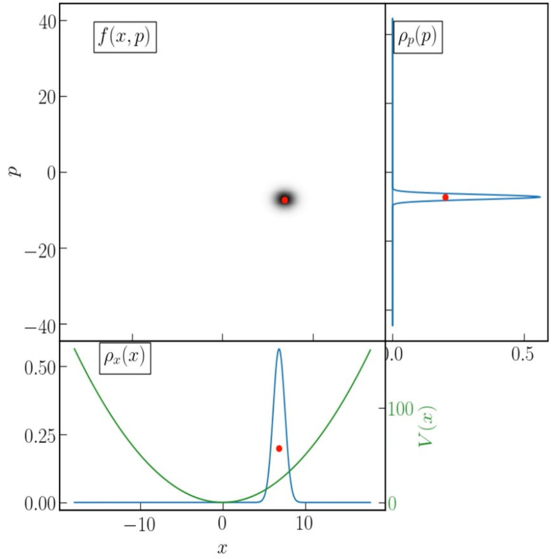
Classical approximation

- In the absence of nonlinearities we would expect this description to survive
- Nonlinearities introduce quantum corrections on some timescale

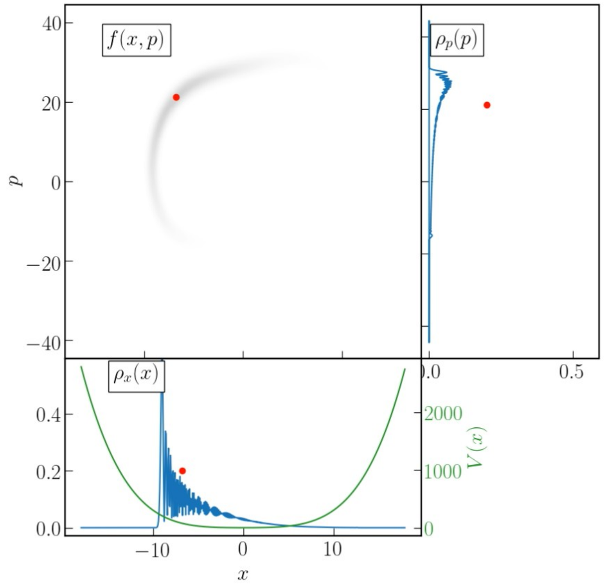


Central Questions

- Which case is relevant for the simulation of ultra light dark matter?



Weakly nonlinear



Strongly nonlinear

Let’s look at an analogous system that contain all the important components of this problem

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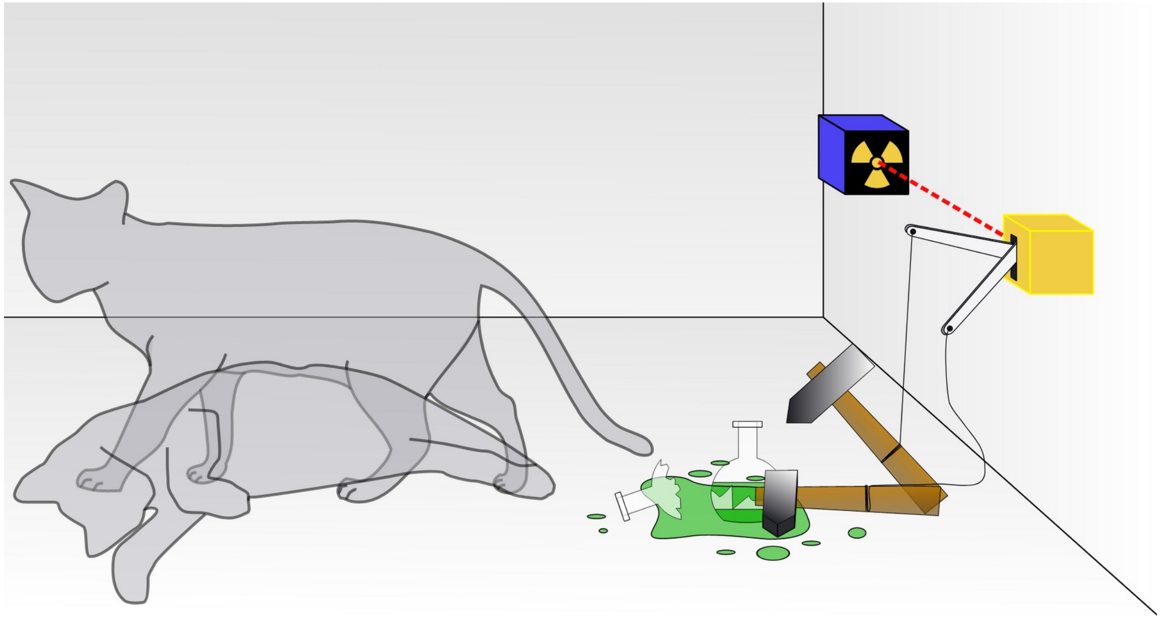

https://en.wikipedia.org/wiki/Schr%C3%B6dinger%27s_cat#/media/File:Schrodingers_cat.svg

Schrödinger’s Cat

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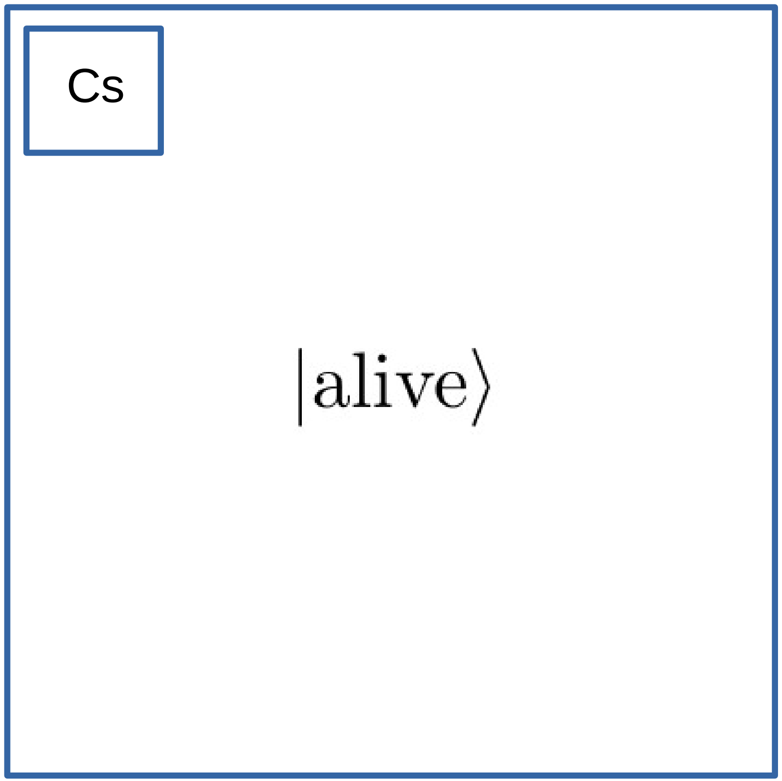
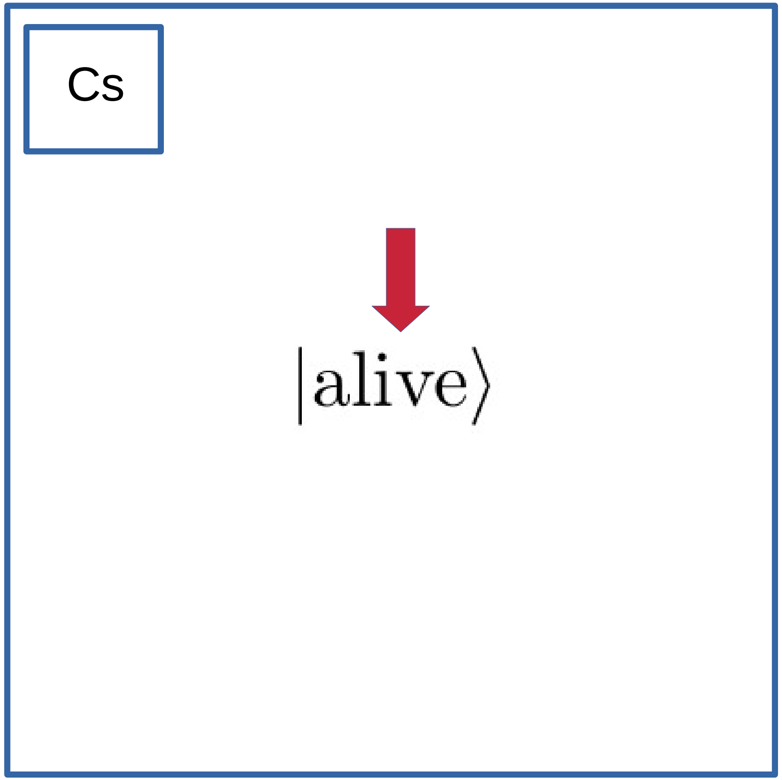


Schrödinger's Cat

$|\psi(t=0)\rangle$

- The system starts in a state well described by classical mechanics



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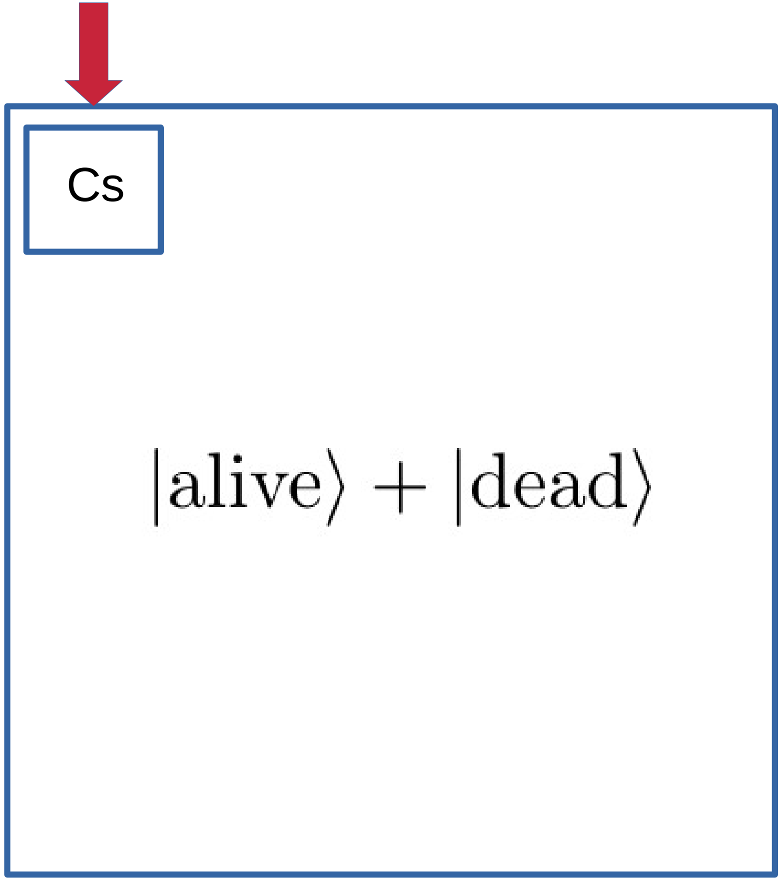
τ_{NL}

- On some timescale nonlinear interactions will create a system poorly described by classical mechanics



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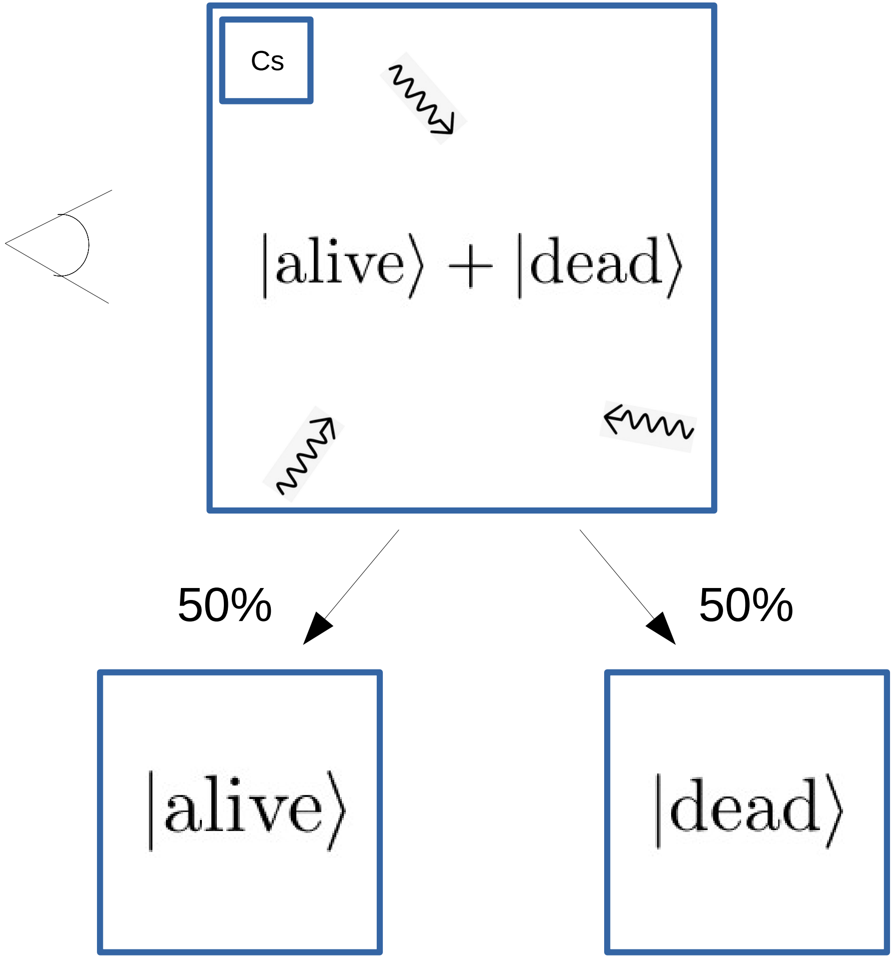
- $|\psi(t=0)\rangle$ The system starts in a state well described by classical mechanics
- τ_{NL} On some timescale nonlinear interactions will create a system poorly described by classical mechanics
- τ_{env} On some timescale environmental interactions ("observers") will send this system to its pointer states



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50% $|\text{alive}\rangle$

50% $|\text{dead}\rangle$

Pointer states

Schrödinger's Cat

Initial conditions

$$|\mathrm{alive}\rangle$$

Life time of Cs

τ_{NL}

Quantum corrections

Quantum state

$$|\mathrm{alive}\rangle + |\mathrm{dead}\rangle$$

Env interaction

τ_{env}

Decoherence

Pointer states

$$|\mathrm{alive}\rangle \quad |\mathrm{dead}\rangle$$

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Schrödinger's Cat

Initial conditions — Quantum state — Pointer states

$$|\mathrm{alive}\rangle \xrightarrow[\tau_{NL}]{\text{Life time of Cs}} |\mathrm{alive}\rangle + |\mathrm{dead}\rangle \xrightarrow[\tau_{\mathrm{env}}]{\text{Env interaction}} |\mathrm{alive}\rangle \quad |\mathrm{dead}\rangle$$

Quantum corrections — Decoherence

Schrödinger's Cat

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$|\mathrm{alive}\rangle$

Life time of Cs

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Schrödinger's Cat

Initial conditions | Quantum state | Pointer states

$|\text{alive}\rangle$ → Life time of Cs, τ_{NL} → $|\text{alive}\rangle + |\text{dead}\rangle$ → Env interaction, τ_{env} → $|\text{alive}\rangle \quad |\text{dead}\rangle$

Quantum corrections | Decoherence | Cat

Initial conditions | Quantum state | Pointer states

$|\text{coherent}\rangle$ → Analytic estimates, τ_{NL} → $|???\rangle$ → Analytic estimates, τ_{env} → $|???\rangle$

Quantum corrections | Decoherence | Dark matter

Schrödinger's Cat

Initial conditions

$|\mathrm{alive}\rangle$

Life time of Cs $\rightarrow$ τ_{NL}

Quantum corrections

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$|\mathrm{alive}\rangle + |\mathrm{dead}\rangle$

Env interaction $\rightarrow$ τ_{env}

Decoherence

Pointer states

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Initial conditions

$|\mathrm{coherent}\rangle$

Analytic estimates $\rightarrow$ τ_{NL}

Quantum corrections

Quantum state

$|???\rangle$

Analytic estimates $\rightarrow$ τ_{env}

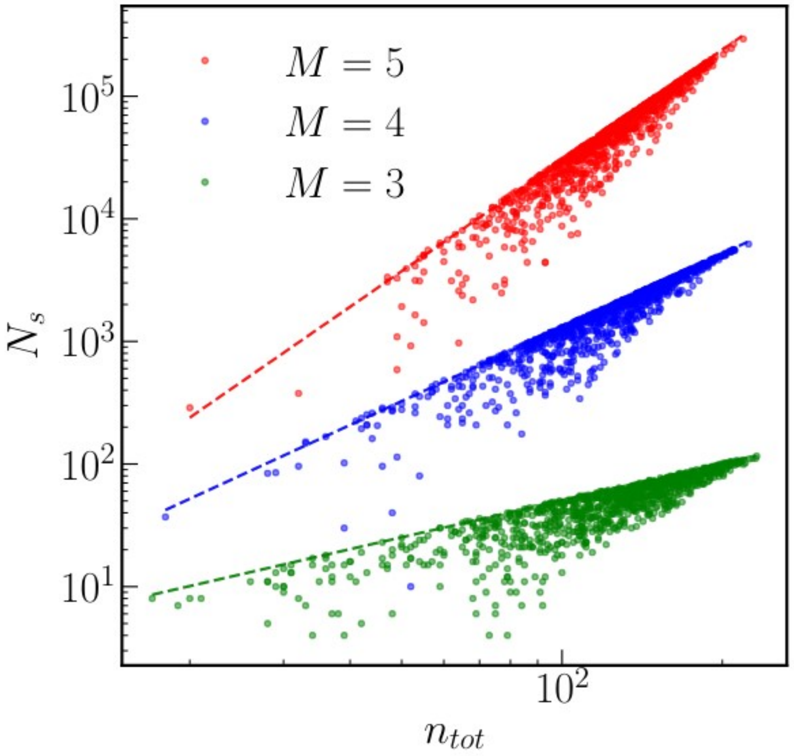
Decoherence

Pointer states

$|???\rangle$

Approaches to answers

- Previous approaches generally separate into two groups
  - Order of magnitude estimates
  - Simulations of small quantum "number eigenstates"
- We directly simulate the evolution of quantum corrections for coherent states on a variety of scales
- Made difficult by the scaling of Quantum Hilbert spaces



Eberhardt et al., PRD (Feb 2022)

Methods: truncated Wigner approximation

- For large systems we want to move to "quantum phase space"

Large Systems:

$$M = 512^3,\ n_{tot} < 10^{14}$$

Methods: truncated Wigner approximation

- For large systems we want to move to "quantum phase space"
- This is done using the Weyl symbol of the quantum state, the Wigner function

Quantum state:

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

Wigner function:

$$\hat{\rho} \rightarrow f[\psi, \psi^*]$$

$|\psi\rangle$ → Go from a vector in Hilbert space to a probability functional on field configurations

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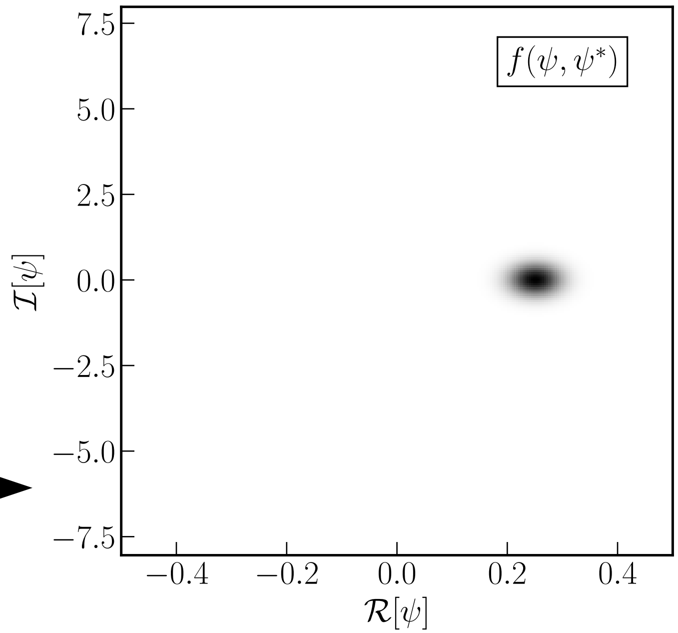

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Von Neumann eqn:

$$\partial_t \hat{\rho} = i[\hat{\rho}, \hat{H}]$$

$$\partial_t f[\psi, \psi^*] = i\,\{\,\{\,f[\psi, \psi^*], H[\psi, \psi^*]\,\}\,\}_m$$

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$$\begin{aligned}\partial_t f[\psi,\psi^*] &= i\,\{\,\{\,f[\psi,\psi^*], H[\psi,\psi^*]\,\}\,\}_m \\ &= i\,\{\,f[\psi,\psi^*], H[\psi,\psi^*]\,\}_p + \mathcal{O}(1/n_{tot})\end{aligned}$$

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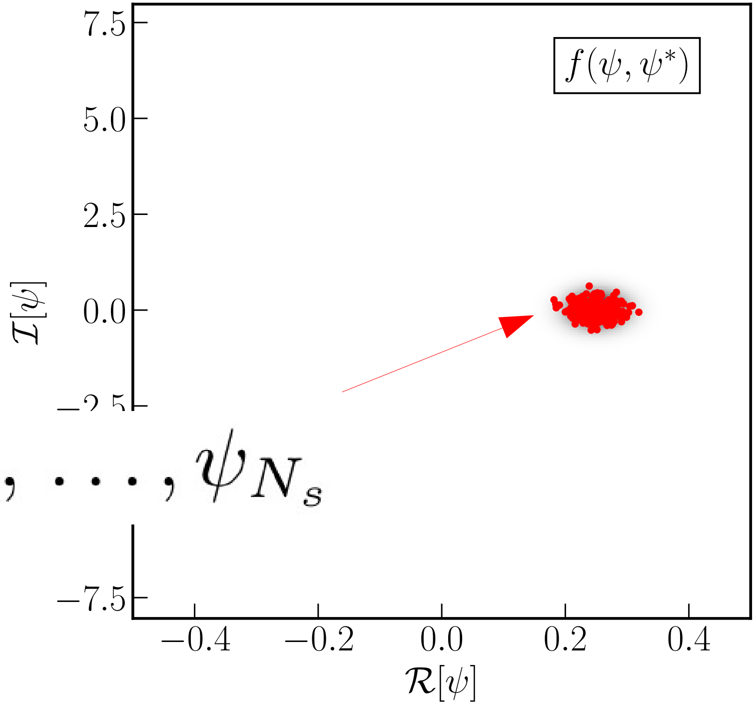
- Approximate Wigner function as ensemble of classical fields drawn from Wigner distribution



$$\psi_1,\ \psi_2,\ \ldots,\ \psi_{N_s}$$

$$f[\psi,\psi^*] = \sum_s \delta[\psi - \psi_s(x)]\,\delta[\psi^* - \psi_s^*(x)]$$

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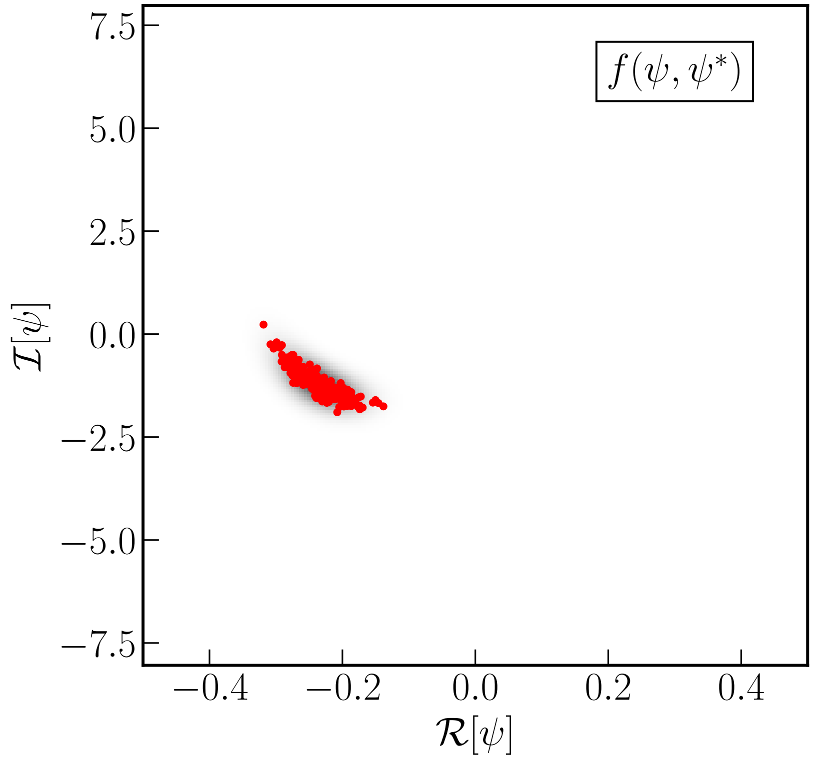

$$\partial_t \psi_s(x) = i \{ \psi_s(x),\ H[\psi_s(x), \psi_s^*(x)] \}_c$$

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$$M = 512^3, \; n_{tot} < 10^{14}$$

$$\partial_t \psi_1(x) = i \{ \psi_1(x), \; H[\psi_1(x), \psi_1^*(x)] \}_c \longrightarrow \text{CPU}$$

$$\partial_t \psi_2(x) = i \{ \psi_2(x), \; H[\psi_2(x), \psi_2^*(x)] \}_c \longrightarrow \text{CPU}$$

$$\cdots$$

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$$\sim \mathcal{O}(N_s M^D \log M)$$

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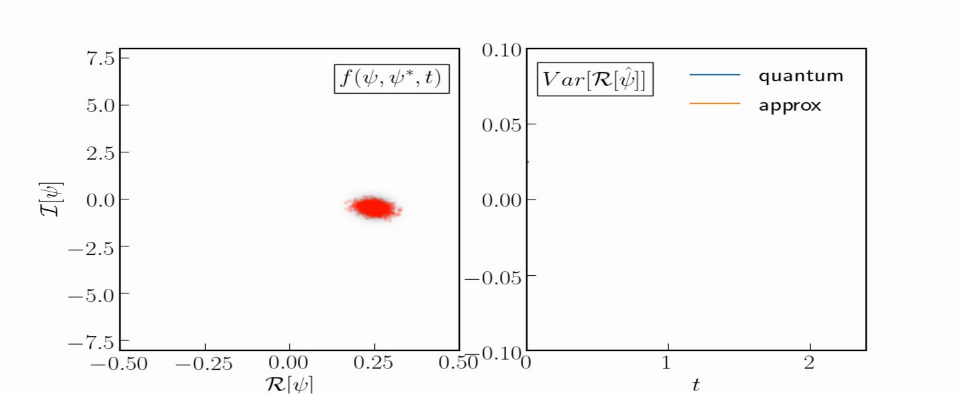
$$\sim \mathcal{O}(N_s M^D \log M)$$

Methods: truncated Wigner approximation

- The truncated Wigner approximation has a lot of good properties:
  - Good scaling with problem size
  - Highly parallelizable
  - Accurate for a long time



Methods: truncated Wigner approximation

- We can also model decoherence

Methods: truncated Wigner approximation

- We can also model decoherence
- We start by defining a system which includes an environment and dark matter component

$$\underbrace{|A\rangle}_{\text{all}} = \overbrace{|\mathrm{DM}\rangle}^{\text{dark matter}} \underbrace{|\mathcal{E}\rangle}_{\text{environment}}$$

Methods: truncated Wigner approximation

- We can also model decoherence
- We start by defining a system which includes an environment and dark matter component
- We then model the dark matter-environment interaction

$$\hat{H}_A = \hat{H}_{\rm DM} + \hat{H}_{\mathcal{E}} + \hat{H}_{\rm int}$$

Gravitational interaction

Methods: truncated Wigner approximation

- We can also model decoherence
- We start by defining a system which includes an environment and dark matter component
- We then model the dark matter-environment interaction
- Use this Hamiltonian and a joint Wigner function to describe the evolution of the system

$$\hat{H}_A = \hat{H}_{\mathrm{DM}} + \hat{H}_{\mathcal{E}} + \hat{H}_{\mathrm{int}}$$

$$f[\psi, \psi^*] = \sum_s \delta[\psi - \psi_s(x)]\, \delta[\psi^* - \psi_s^*(x)]$$

$$f_p[p, r] = \sum_{si} \delta[r - r_i^s(t)]\, \delta[p - p_i^s(t)]$$

$$f_{\mathrm{all}} = f_{\mathrm{DM}}[\psi, \psi^*] f_p[p, r],$$

Test particle phase space position initially localized at (r_i, p_i)

Methods: truncated Wigner approximation

- We can also model decoherence
- We start by defining a system which includes an environment and dark matter component
- We then model the dark matter-environment interaction
- Use this Hamiltonian and a joint Wigner function to describe the evolution of the system
- Because we know luminous matter has well defined phase space trajectories we know that the decoherence rate must be at least as fast as the test particle enters into a macroscopic super position in phase space

Results

Schrödinger's Cat

Initial conditions — Quantum state — Pointer states

$|\text{alive}\rangle$ → Life time of Cs, τ_{NL} (Quantum corrections) → $|\text{alive}\rangle + |\text{dead}\rangle$ → Env interaction, τ_{env} (Decoherence) → $|\text{alive}\rangle \quad |\text{dead}\rangle$

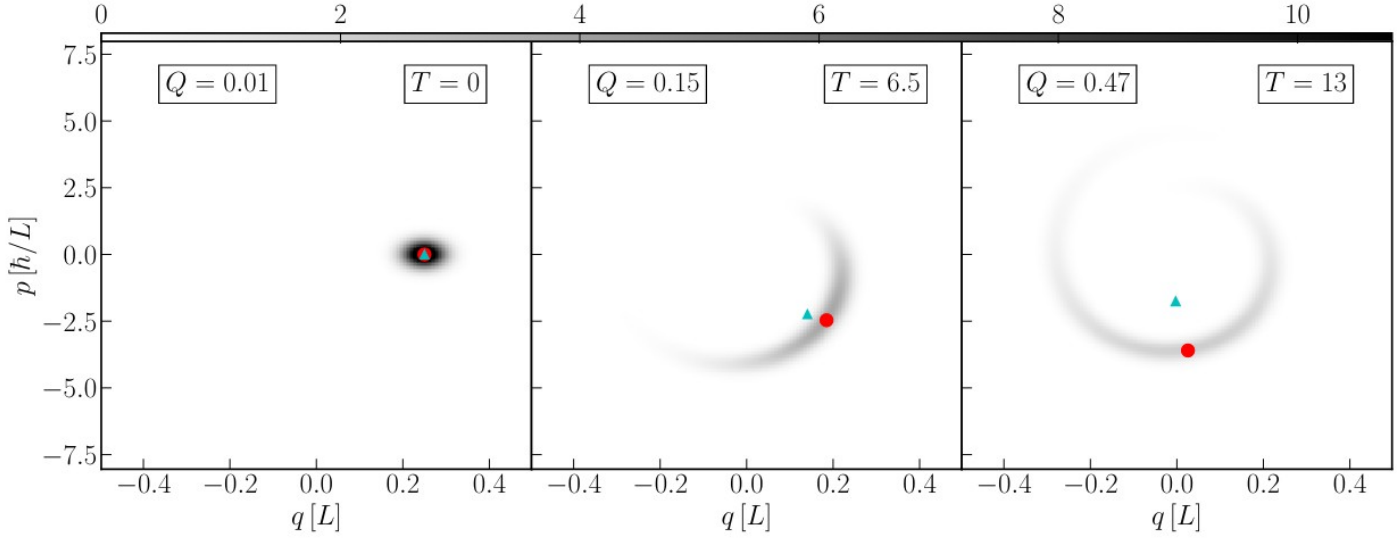
Initial conditions — Quantum state — Pointer states

$|\text{coherent}\rangle$ → Analytic estimates, τ_{NL} (Quantum corrections) → $|???\rangle$ → Analytic estimates, τ_{env} (Decoherence) → $|???\rangle$

Results

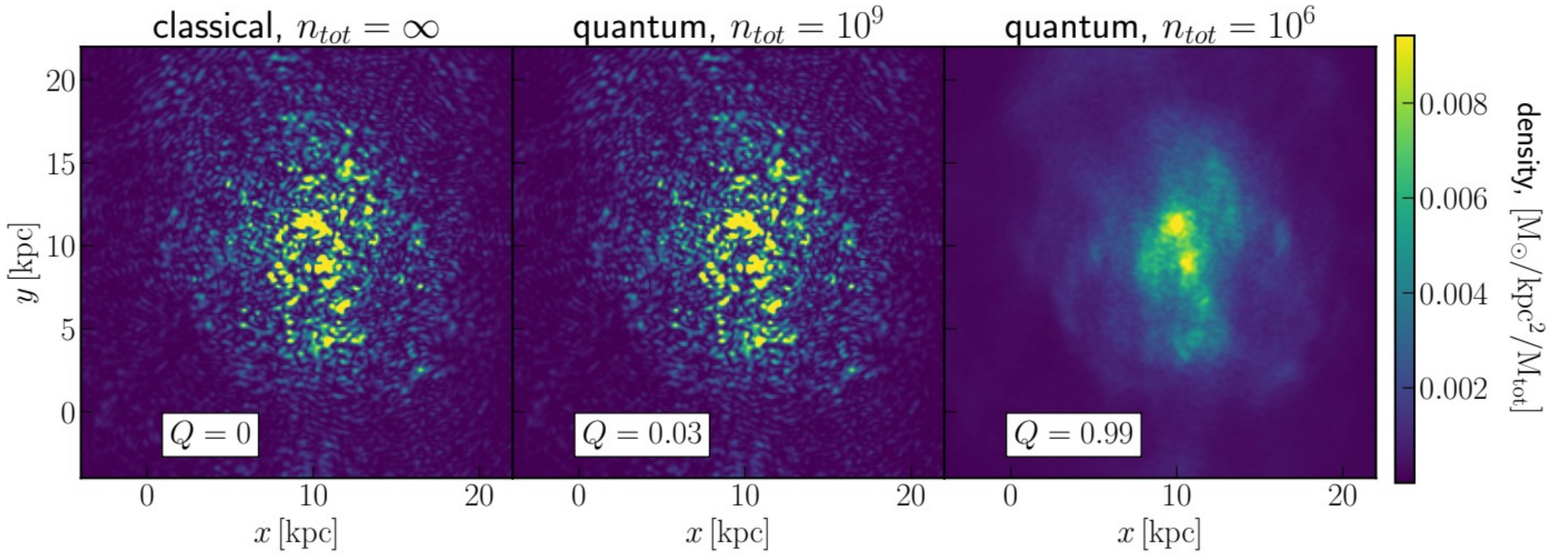
- We quantify the size of corrections using a parameter Q which measures how the average spread in the wavefunction compares to the mean value
- Q goes from 0 to 1 in all systems

$$Q = \frac{1}{n_{tot}} \int dx \, \langle \delta\hat{\psi}^\dagger(x) \delta\hat{\psi}(x) \rangle$$



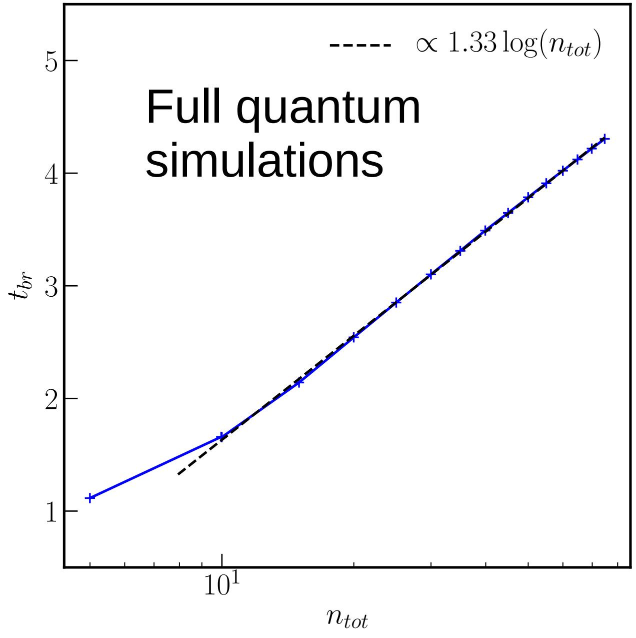
Results

- We quantify the size of corrections using a parameter Q which measures how the average spread in the wavefunction compares to the mean value
- Not a unique choice (or only one we looked at) but reliable indicator of differences between quantum and classical evolutions



Results

- First analysis we performed was to test how long it takes for Q to grow to a certain size (this defined the **quantum breaktime**) as a function of the total number of particles keeping the mean field evolution fixed



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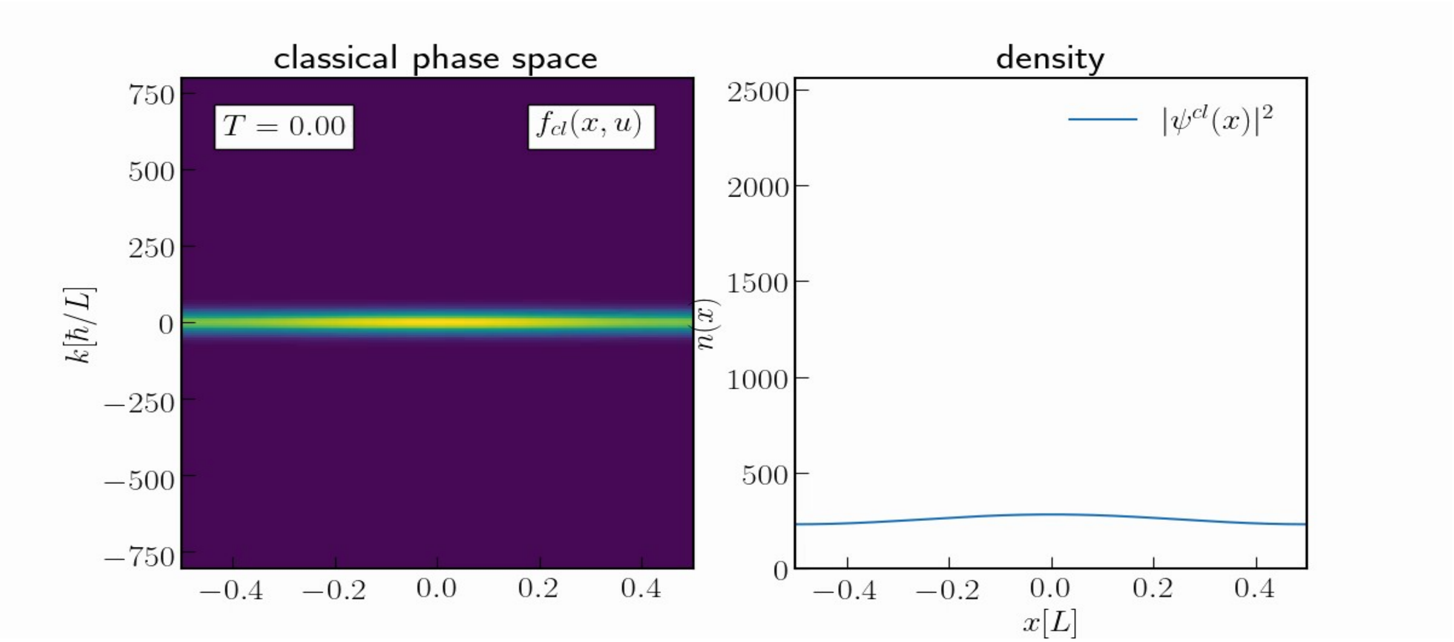

classical phase space
$T = 0.00$
$f_{cl}(x,u)$
$k[\hbar/L]$
750
500
250
0
−250
−500
−750
−0.4
−0.2
0.0
0.2
0.4
density
$|\psi^{cl}(x)|^2$
$n(x)$
2500
2000
1500
1000
500
0
$x[L]$

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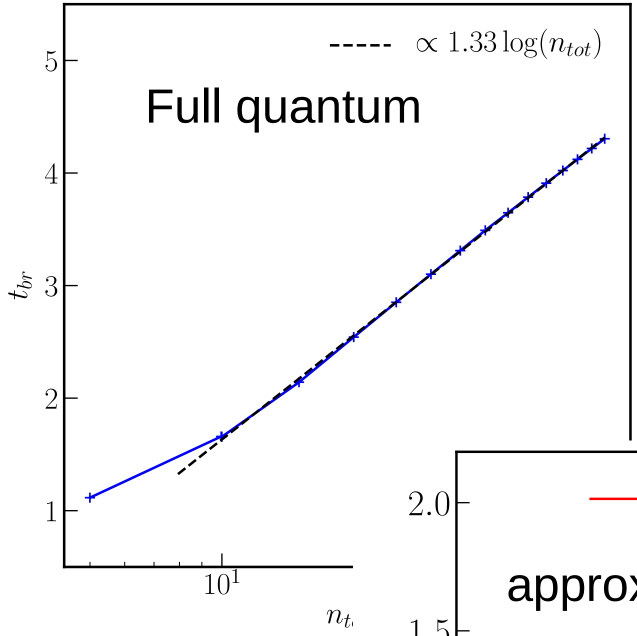
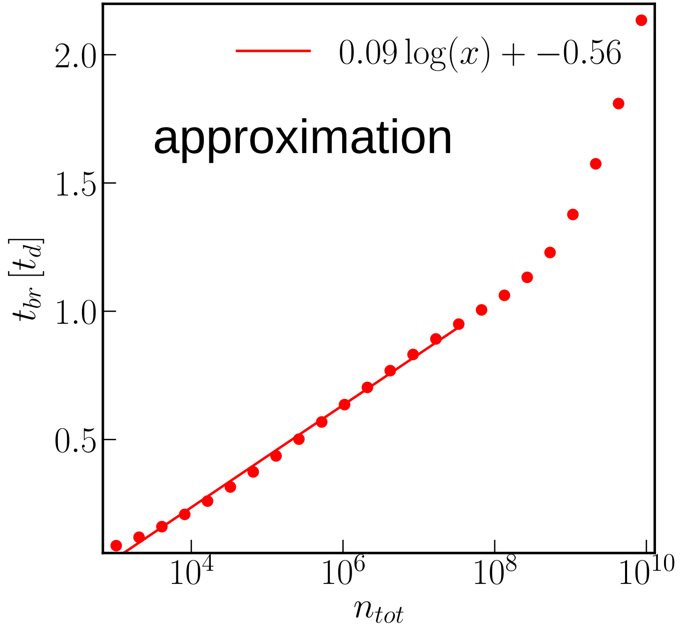
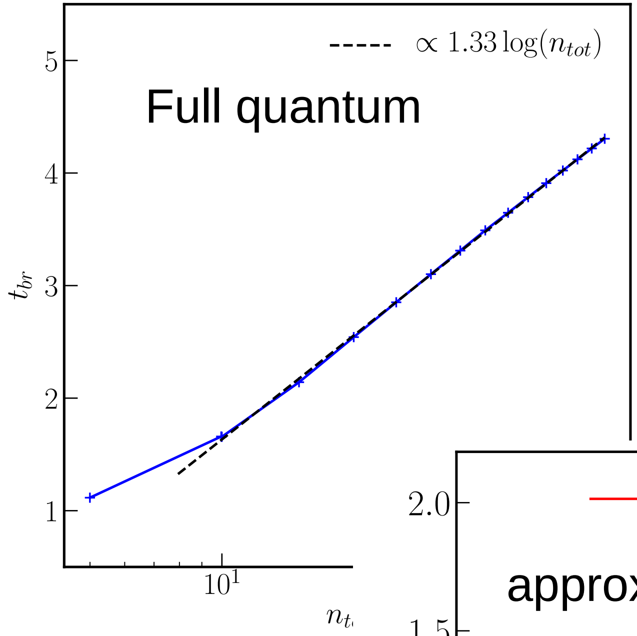
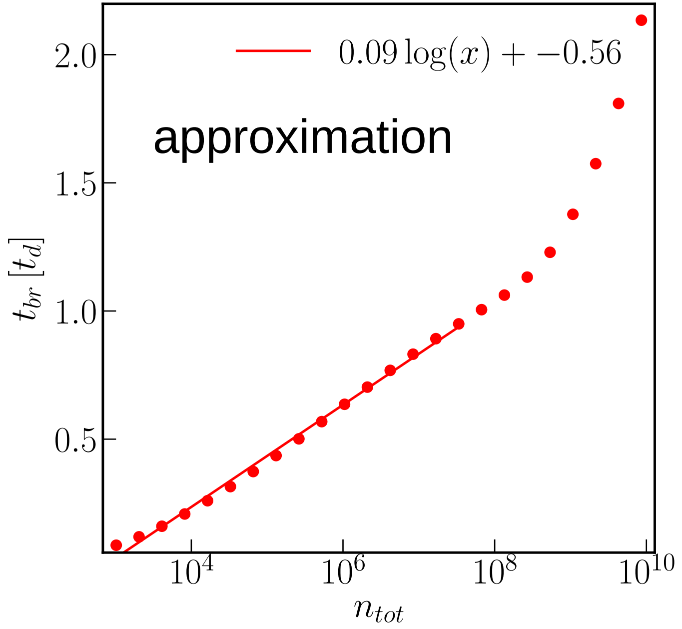




Results

- First analysis we performed was to test how long it takes for Q to grow to a certain size (this defined the **quantum breaktime**) as a function of the total number of particles keeping the mean field evolution fixed
- See a logarithmic enhancement in the breaktime with particle number





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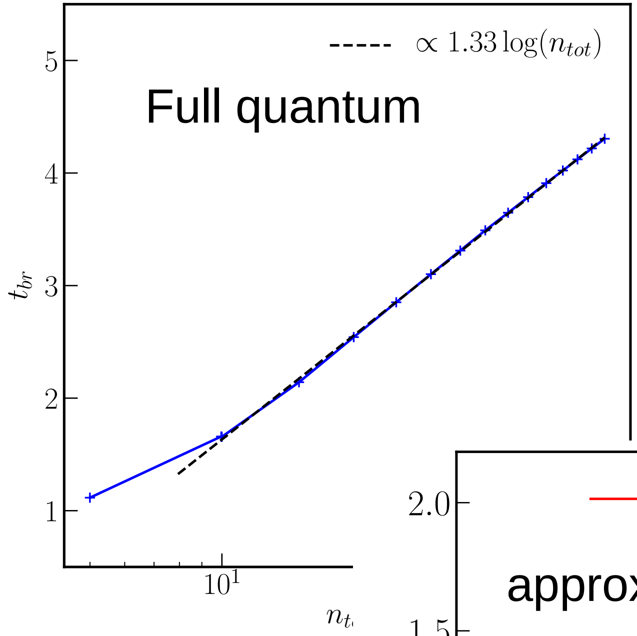

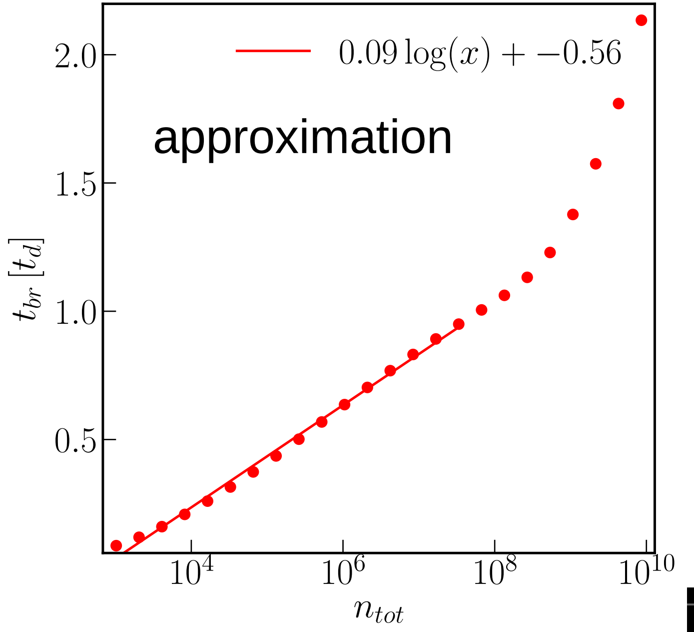

Results

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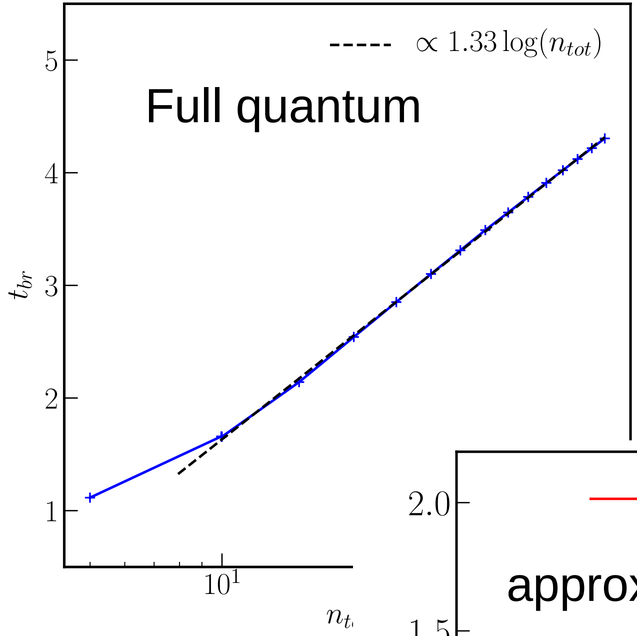
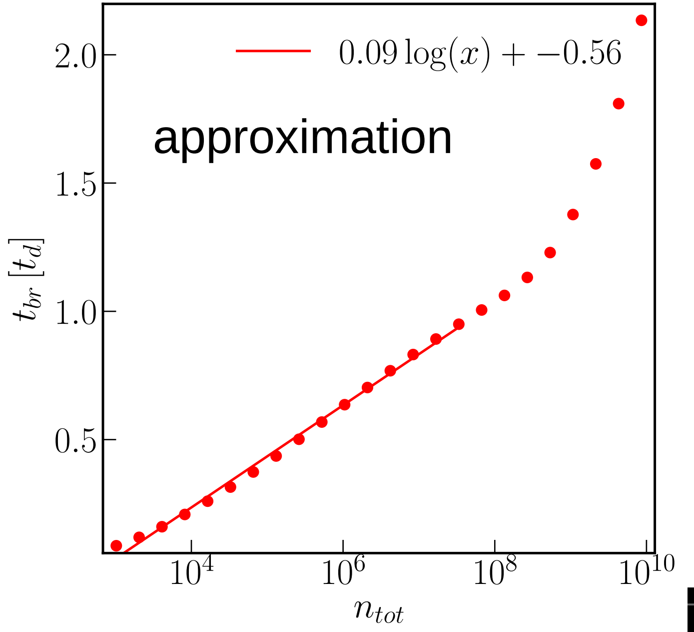
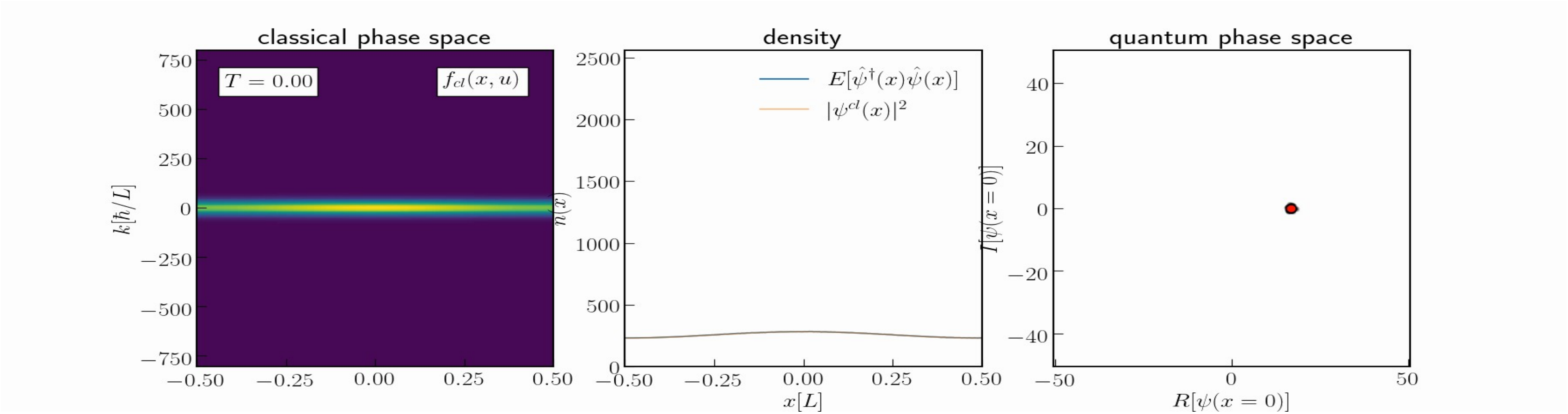
- See a logarithmic enhancement in the breaktime with particle number
- Well known prediction for systems that exhibit classical chaos
- Straightforward to understand in the truncated Wigner context





Results

- Small quantum perturbations in initial conditions spread exponentially



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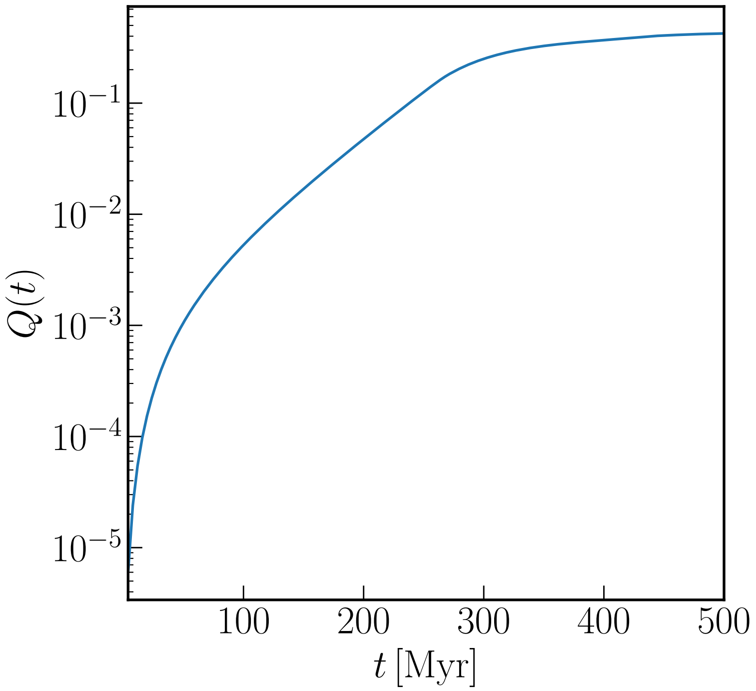

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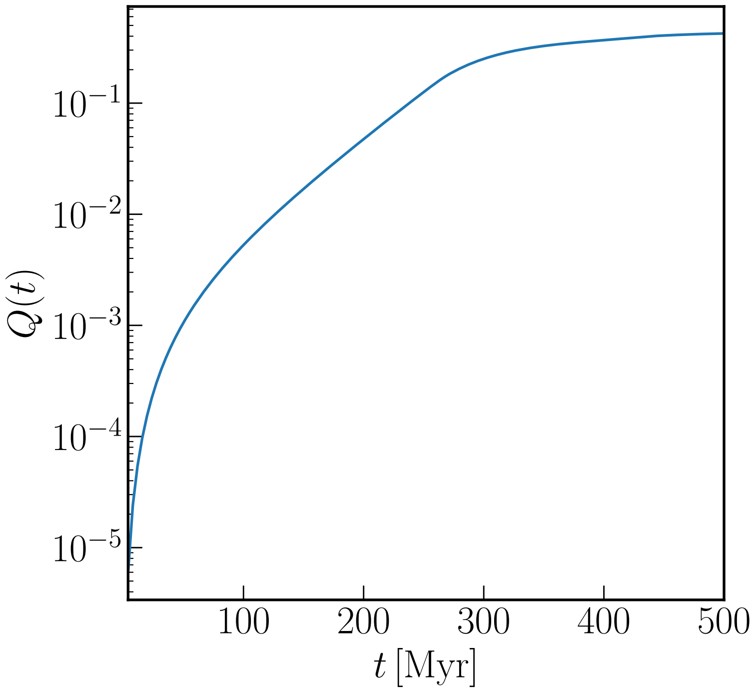
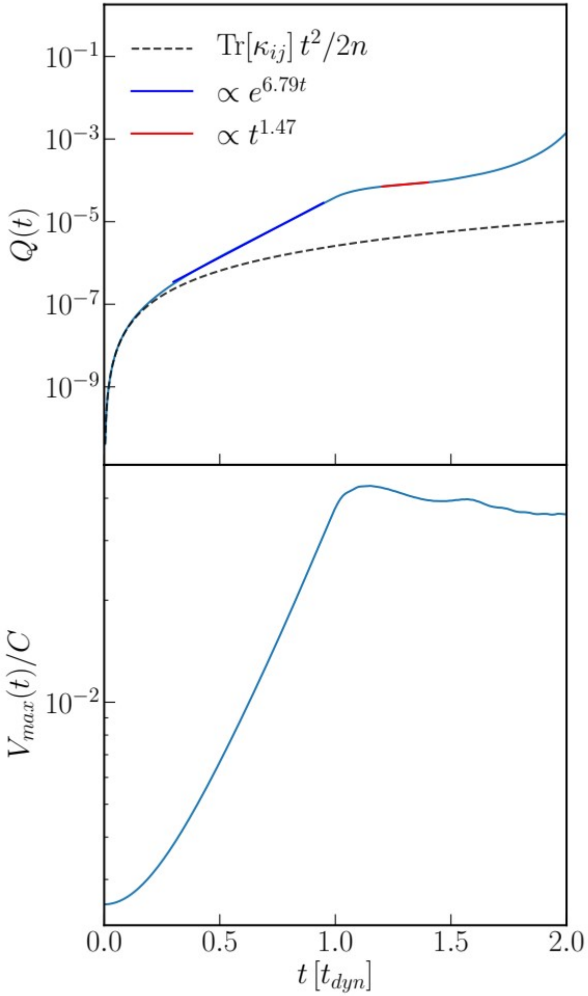
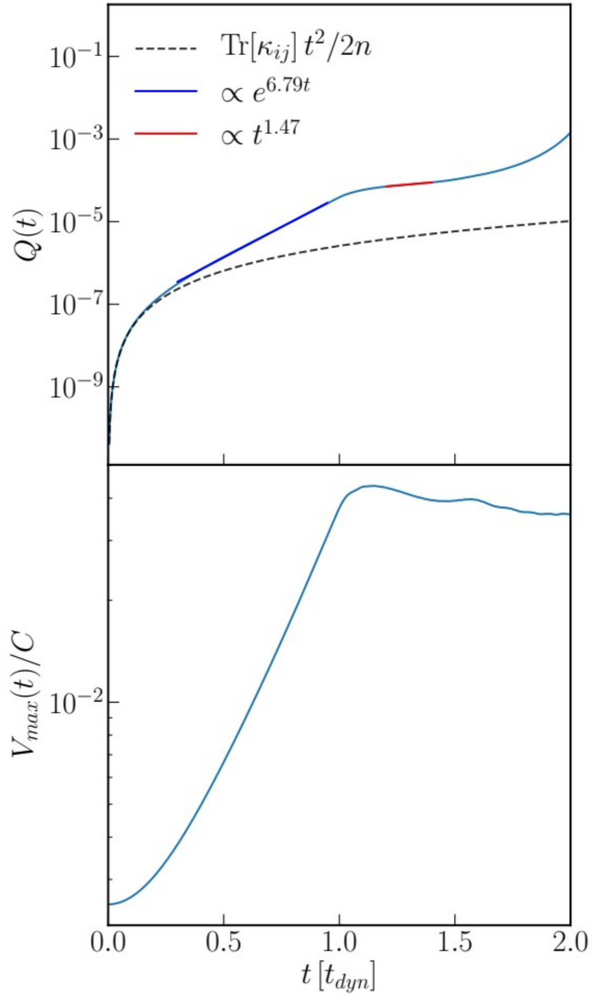




Results

- Second analysis is to look at how Q grows
- Staged growth

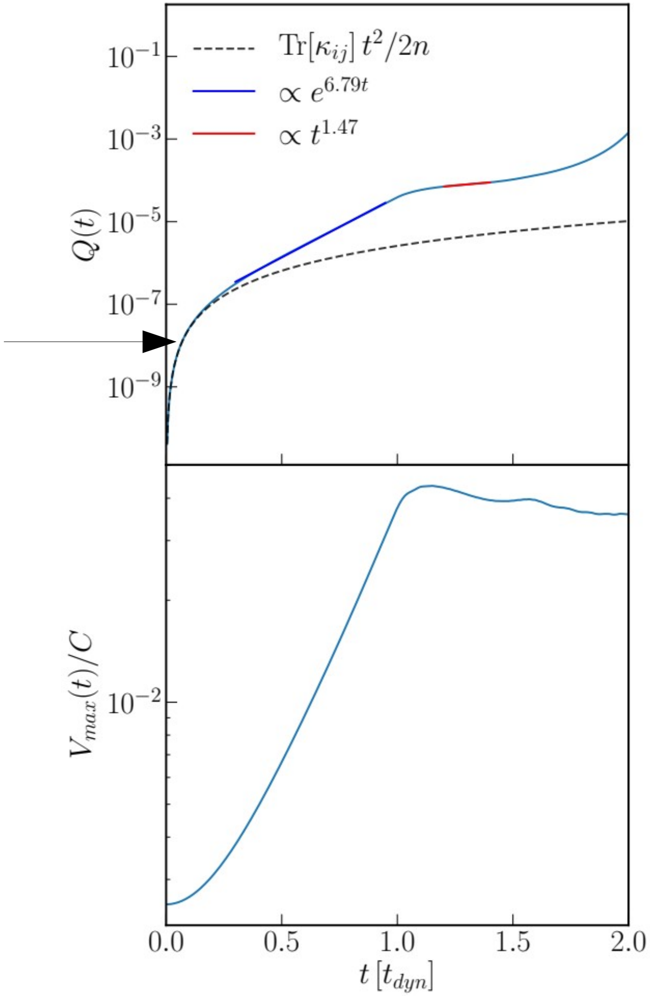


Results

- Second analysis is to look at how Q grows
- Staged growth
  - Initial quadratic growth

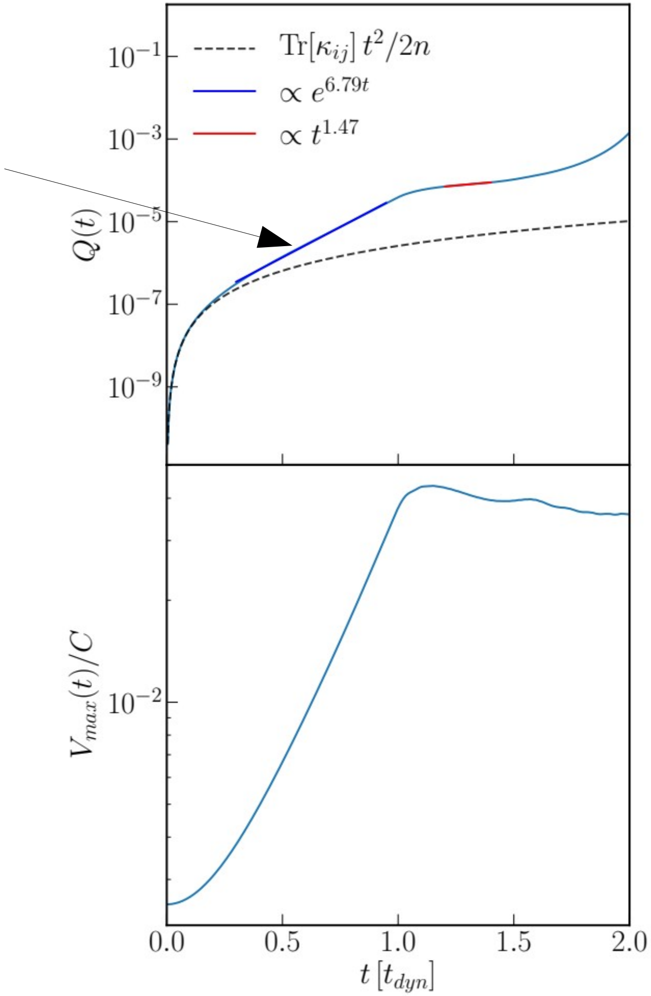
$$\partial_{tt} \langle \delta\hat{a}_i^\dagger \delta\hat{a}_j \rangle \sim 2\mathbb{R} \left[\sum_{kplbc} \Lambda_{pl}^{ij} \Lambda_{bc}^{kj} \langle \hat{a}_b \rangle \langle \hat{a}_c \rangle \langle \hat{a}_p^\dagger \rangle \langle \hat{a}_l^\dagger \rangle \right]$$

$$\equiv \kappa_{ij}$$



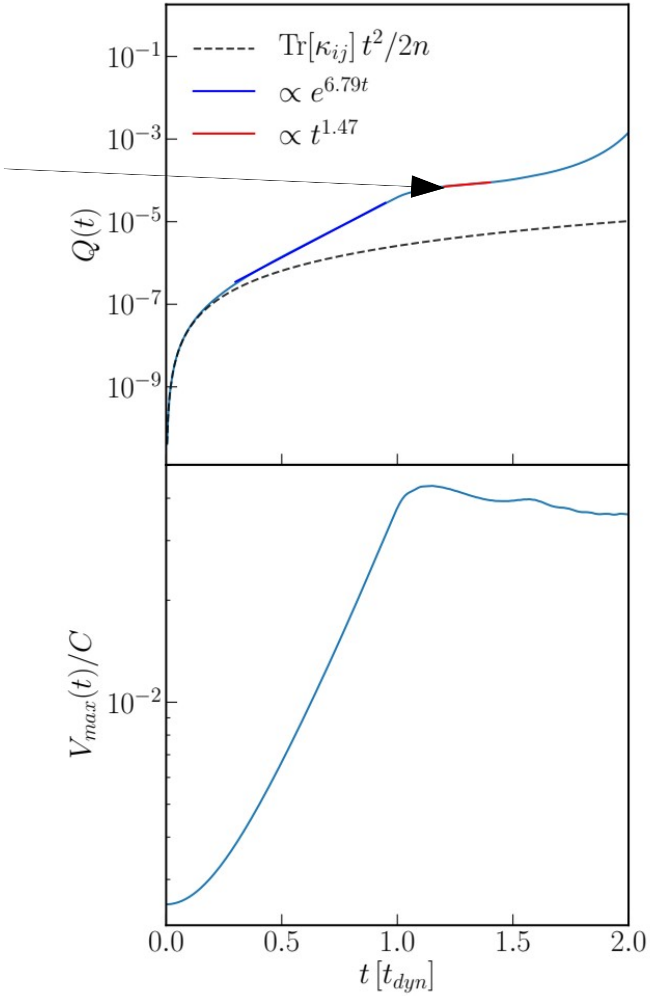
Results

- Second analysis is to look at how Q grows
- Staged growth
  - Initial quadratic growth
  - Exponential growth during collapse



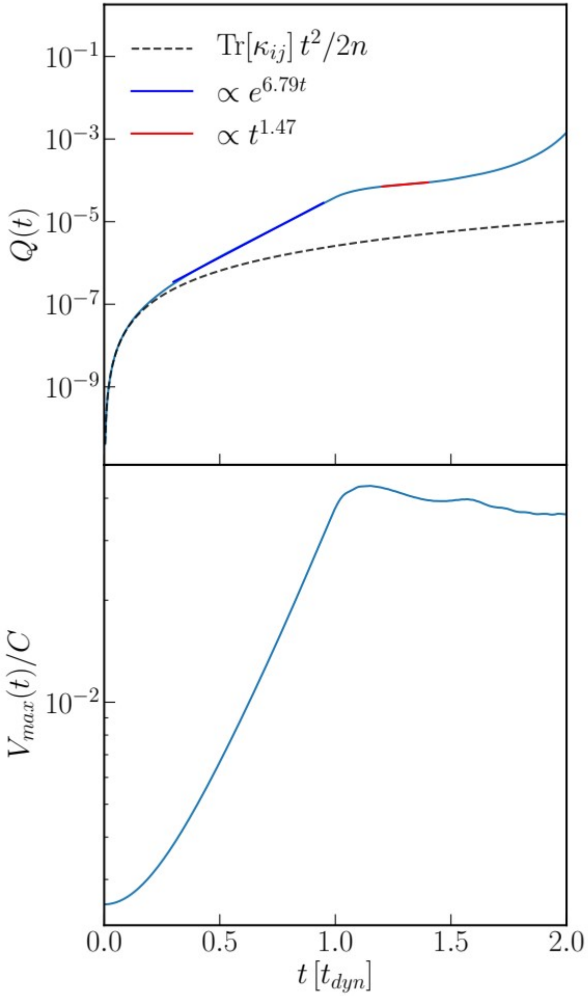
Results

- Second analysis is to look at how Q grows
- Staged growth
  - Initial quadratic growth
  - Exponential growth during collapse
  - Powerlaw after collapse



Results

- Second analysis is to look at how Q grows
- Staged growth
  - Initial quadratic growth
  - Exponential growth during collapse
  - Powerlaw after collapse
- Any powerlaw growth is too slow but exponential growth may be a problem for the classical theory



Results

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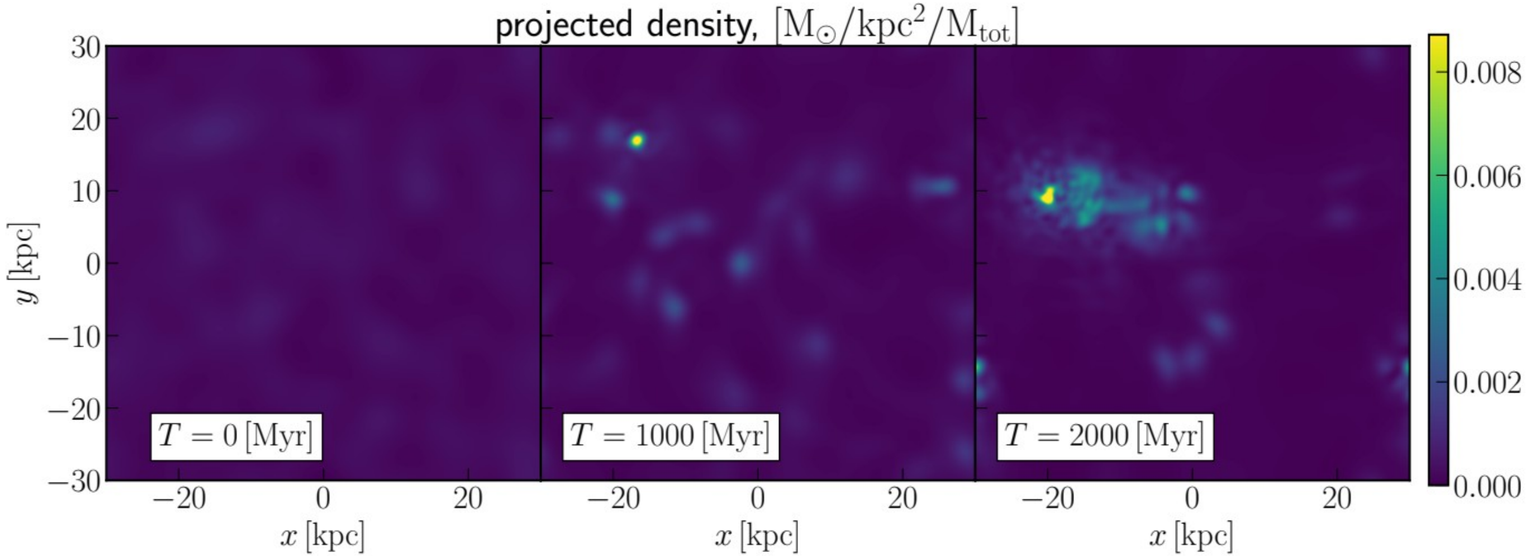

Results

- How does behavior generalize to 3D systems?
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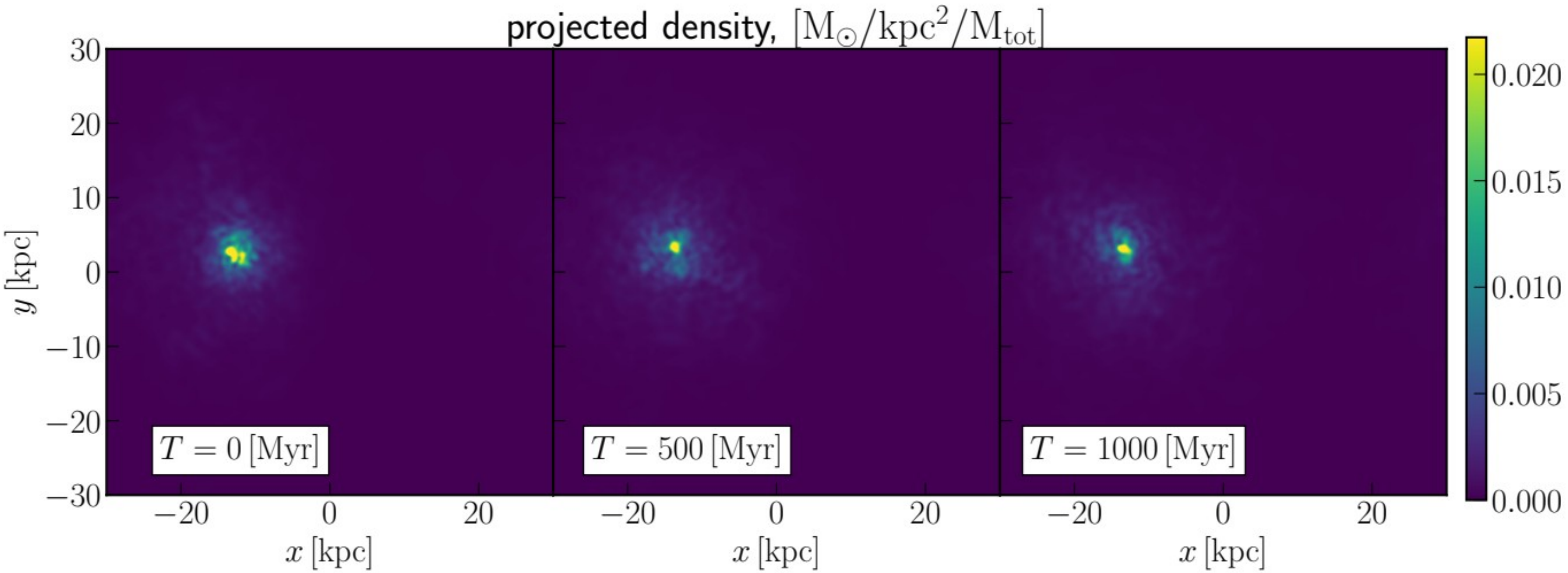
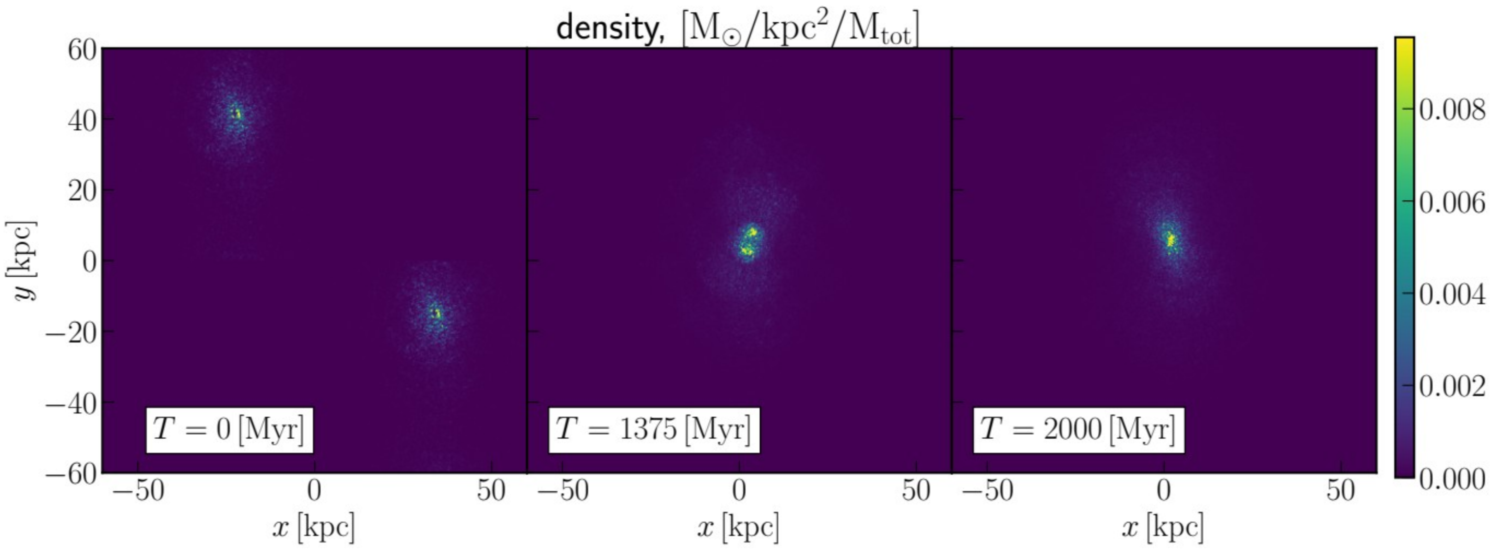


Results

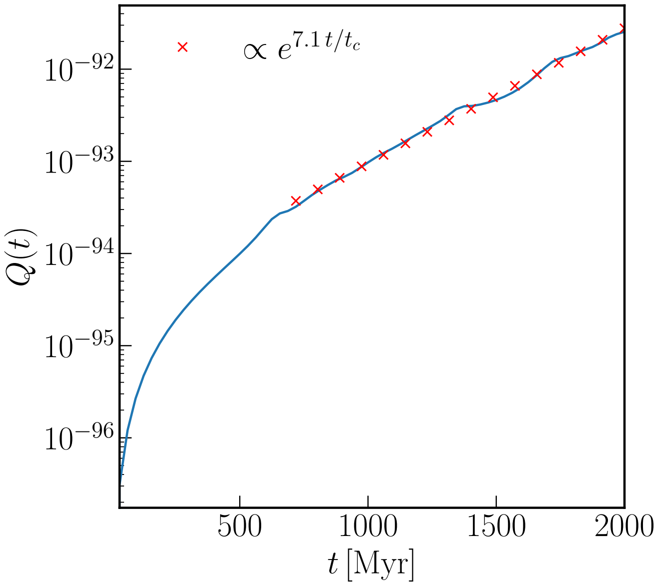
- How does behavior generalize to 3D systems?
- Used 3 test problems: collapse of a random field, stable collapsed object, **merging of two collapsed objects**



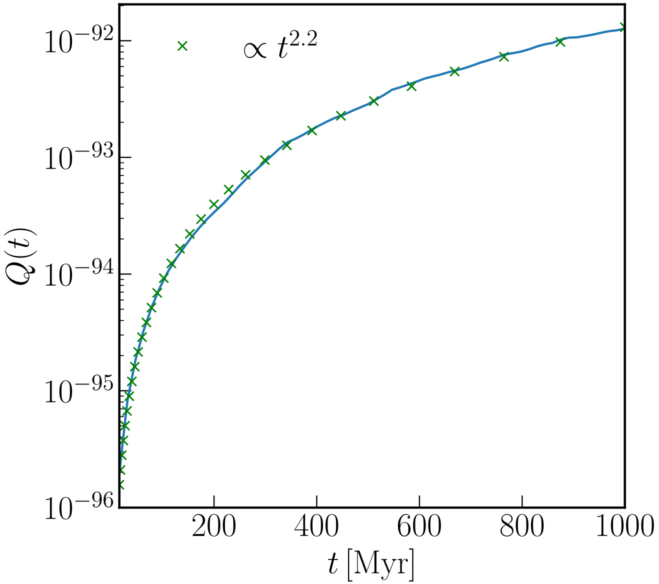
Results

- Results corroborate 1D expectations

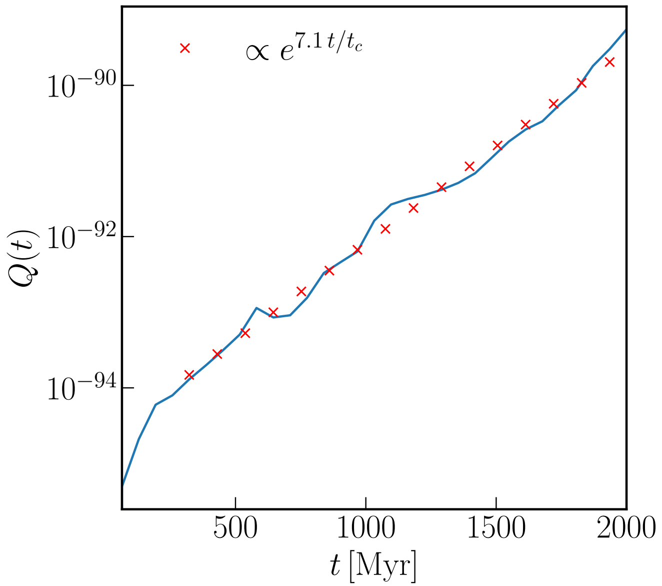
Collapsing



Already collapsed

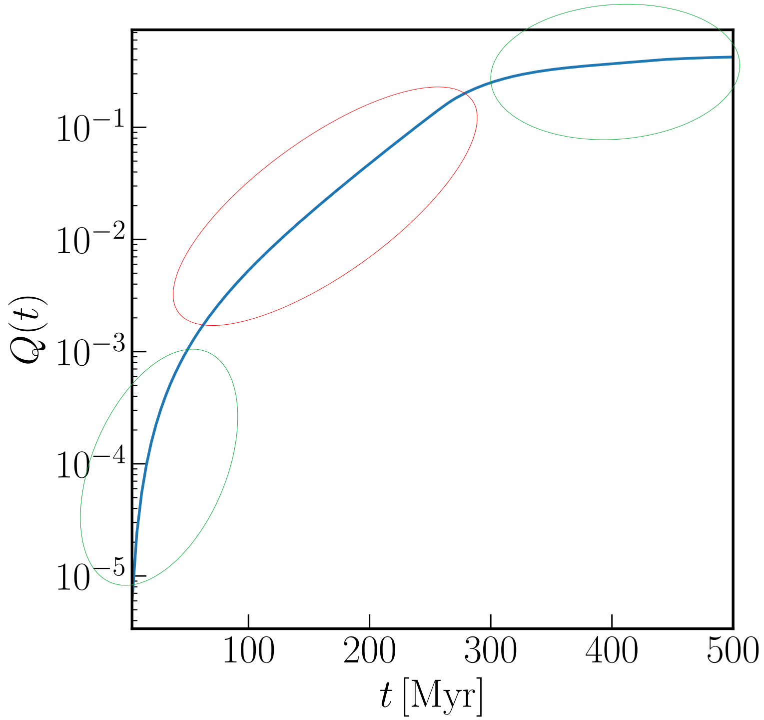


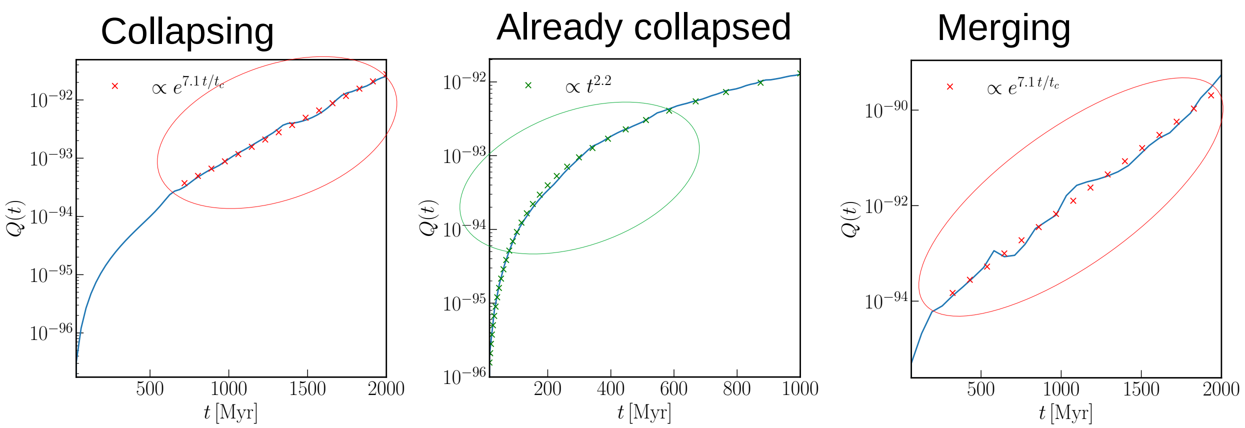
Merging



Results

- Results corroborate 1D expectations
  - Nonlinear collapse/merging is exponential
  - Powerlaw very early and post collapse





3D

Schrödinger's Cat

Initial conditions → **Quantum state** → **Pointer states**

$|\text{alive}\rangle$ — Life time of Cs, τ_{NL} (Quantum corrections) → $|\text{alive}\rangle + |\text{dead}\rangle$ — Env interaction, τ_{env} (Decoherence) → $|\text{alive}\rangle \quad |\text{dead}\rangle$

Initial conditions → **Quantum state** → **Pointer states**

$t_{br} \sim \frac{\ln(n_{tot})}{7} t_d$

$|\text{coherent}\rangle$ — ~~Analytic estimates~~, τ_{NL} (Quantum corrections) → $|???\rangle$ — Analytic estimates, τ_{env} (Decoherence) → $|???\rangle$

Schrödinger's Cat

Initial conditions

$|\text{alive}\rangle$

Life time of Cs

τ_{NL}

Quantum corrections

Quantum state

$|\text{alive}\rangle + |\text{dead}\rangle$

Env interaction

τ_{env}

Decoherence

Pointer states

$|\text{alive}\rangle \quad |\text{dead}\rangle$

Initial conditions

$|\text{coherent}\rangle$

$t_{br} \sim \frac{\ln(n_{tot})}{7} t_d$

~~Analytic estimates~~

τ_{NL}

Quantum corrections

Quantum state

$|???\rangle$

Analytic estimates

τ_{env}

Decoherence

Pointer states

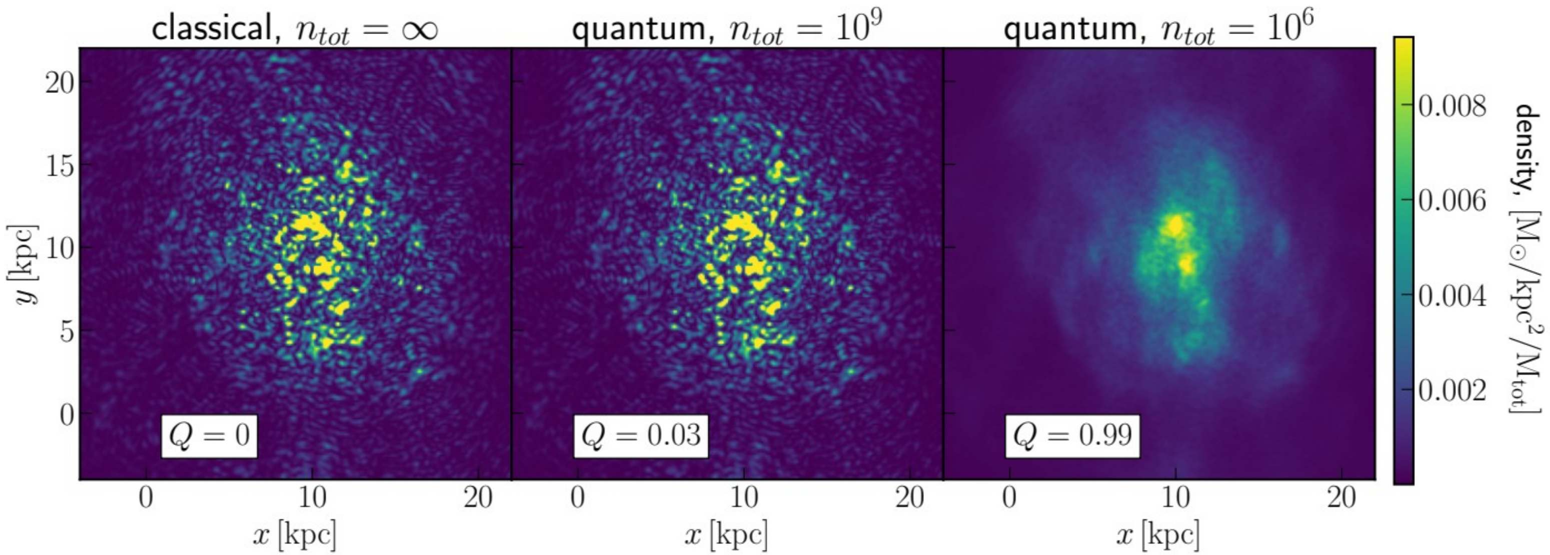
$|???\rangle$

Results

- What predictions do quantum corrections effect?

Results

- Leading order effect is to remove density fluctuations from interference



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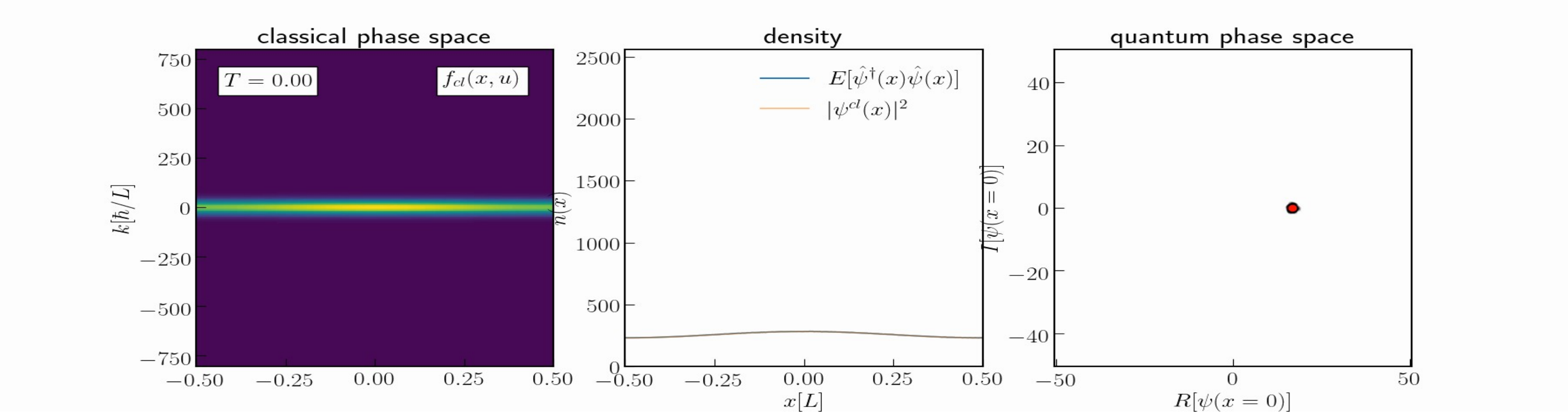

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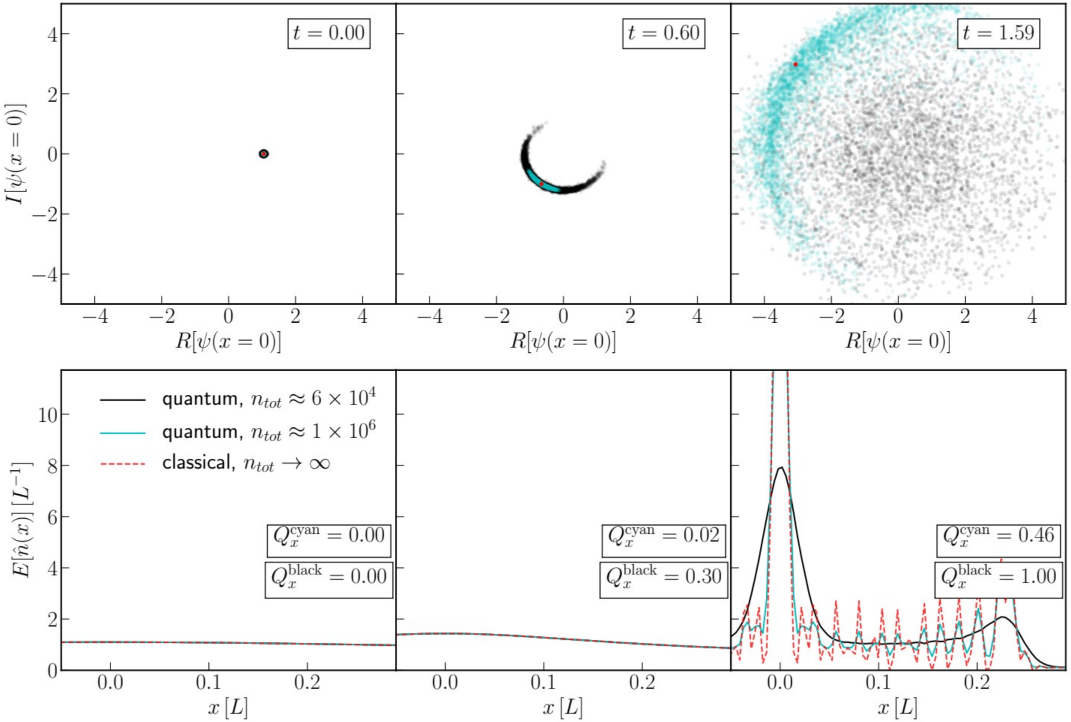


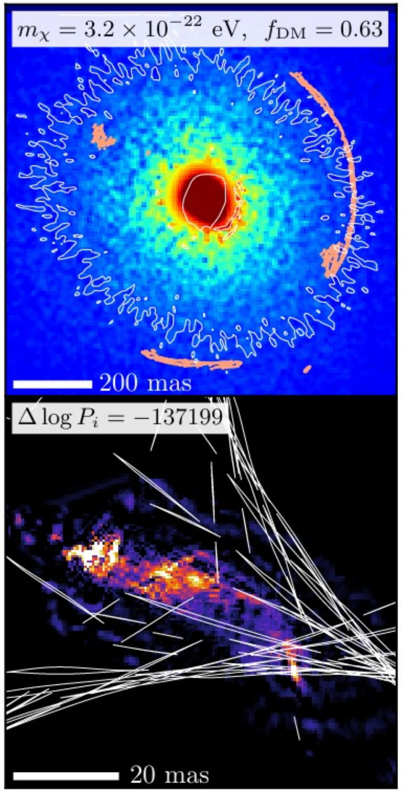
Results

- Leading order effect is to remove density fluctuations from interference
- Effects constraints that rely on the granularity of the density profile
  - Heating of ultra faint dwarf stellar dispersions
  - Constraints from gravitational lensing

Stellar dispersions

$$\Delta\sigma^2 \propto \delta\rho^2$$

Gravitational lensing



Power, et al. MNRAS 2023

Schrödinger's Cat

<u>Initial conditions</u>

$|\text{alive}\rangle$

Life time of Cs

τ_{NL}

<u>Quantum corrections</u>

<u>Quantum state</u>

$|\text{alive}\rangle + |\text{dead}\rangle$

Env interaction

τ_{env}

<u>Decoherence</u>

<u>Pointer states</u>

$|\text{alive}\rangle \quad |\text{dead}\rangle$

<u>Initial conditions</u>

$|\text{coherent}\rangle$

$t_{br} \sim \frac{\ln(n_{tot})}{7} t_d$

Simulated results

τ_{NL}

<u>Quantum corrections</u>

<u>Quantum state</u>

Less granular density

Analytic estimates

τ_{env}

<u>Decoherence</u>

<u>Pointer states</u>

$|???\rangle$

Schrödinger's Cat

Initial conditions — $|\text{alive}\rangle$

→ Life time of Cs, τ_{NL} (**Quantum corrections**) →

Quantum state — $|\text{alive}\rangle + |\text{dead}\rangle$

→ Env interaction, τ_{env} (**Decoherence**) →

Pointer states — $|\text{alive}\rangle \quad |\text{dead}\rangle$

Initial conditions — $|\text{coherent}\rangle$

$t_{br} \sim \frac{\ln(n_{tot})}{7} t_d$

→ Simulated results, τ_{NL} (**Quantum corrections**) →

Quantum state — Less granular density

→ Analytic estimates, τ_{env} (**Decoherence**) →

Pointer states — $|???\rangle$

Results

- Test decoherence by coupling our dark matter state to a test particle

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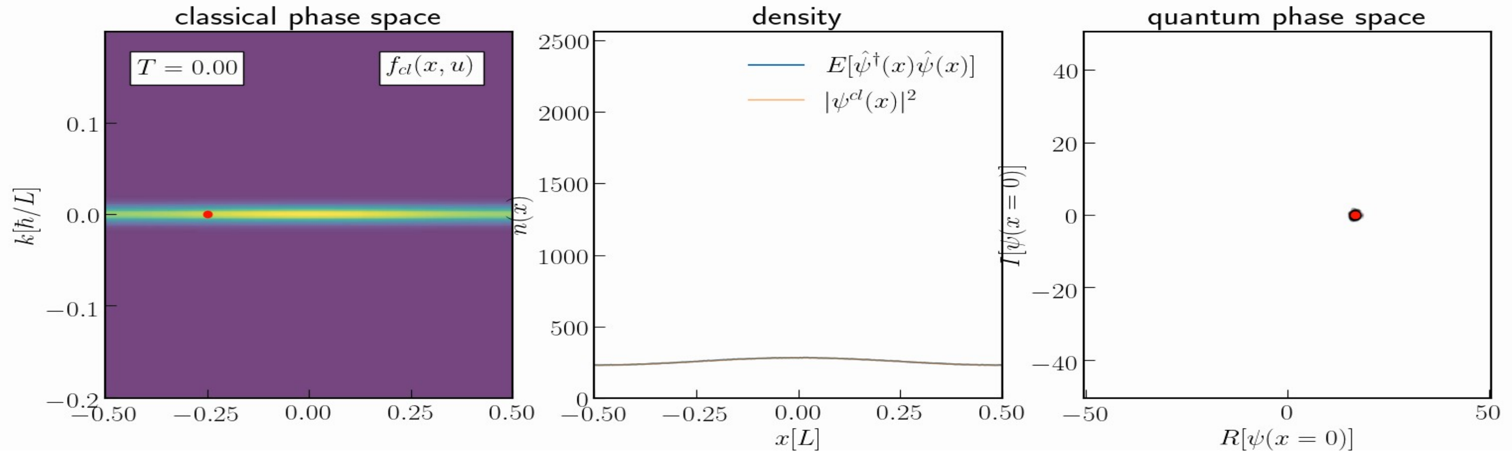
- Over time the test particle will evolve in a super position on phase space



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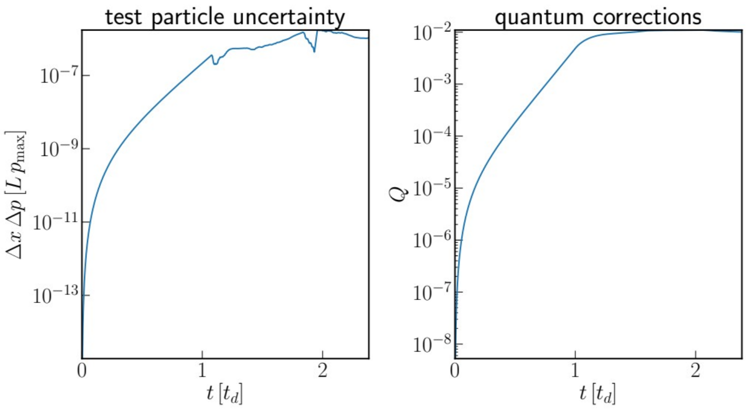

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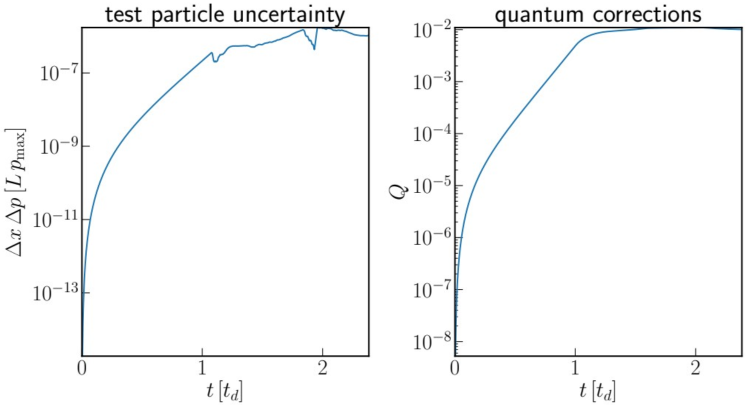

Results

- Test decoherence by coupling our dark matter state to a test particle
- Over time the test particle will evolve in a super position on phase space
- This occurs at the same rate as quantum corrections are introduced
- Unlike Schrodinger’s cat, both the quantum corrections and the decoherence are caused by the same thing, gravity
- Difficult to evolve into a state with large quantum corrections without also putting test particles into macroscopic super positions which we do not observe
- The decoherence time scale must be at least as fast as the nonlinear timescale

Schrödinger's Cat

Initial conditions

$|\text{alive}\rangle$

Life time of Cs

τ_{NL}

Quantum corrections

Quantum state

$|\text{alive}\rangle + |\text{dead}\rangle$

Env interaction

τ_{env}

Decoherence

Pointer states

$|\text{alive}\rangle \quad |\text{dead}\rangle$

Initial conditions

$|\text{coherent}\rangle$

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Simulated results

τ_{NL}

Quantum corrections

Quantum state

Less granular density

$\tau_{\text{env}} \lesssim \tau_{\text{NL}}$

~~Analytic estimates~~

τ_{env}

Decoherence

Pointer states

$|???\rangle$

Schrödinger's Cat

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Life time of Cs

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<u>Quantum corrections</u>

<u>Quantum state</u>

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<u>Decoherence</u>

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Simulated results

τ_{NL}

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Simulated results

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Conclusions

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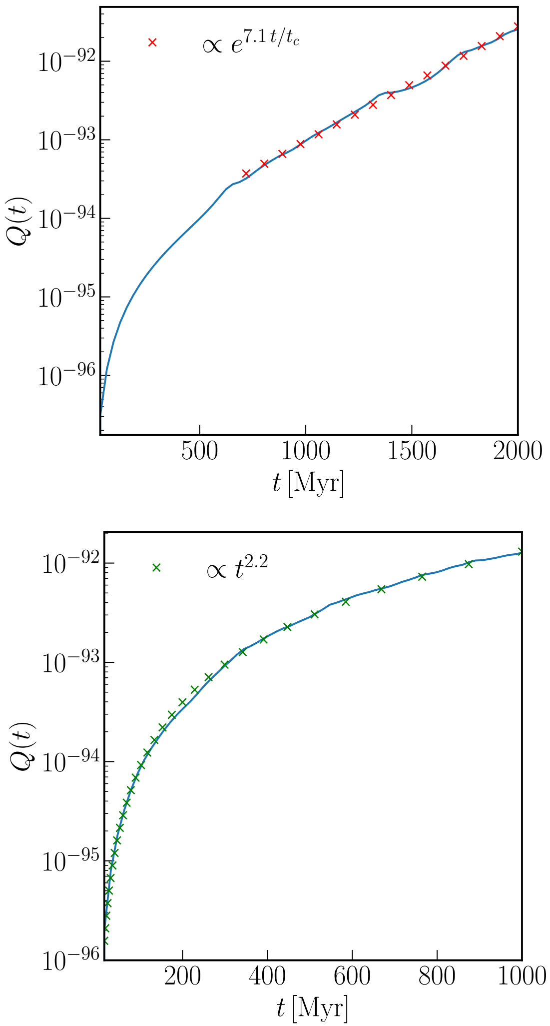
- Quantum corrections:
  - grow exponentially in systems that are experience nonlinear growth (collapsing, merging, etc)
  - Grow slowly in systems already collapsed systems



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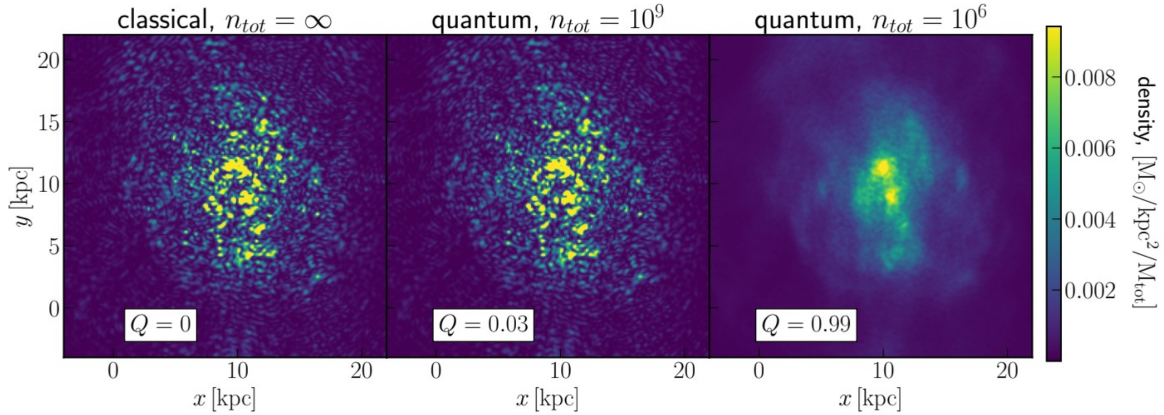

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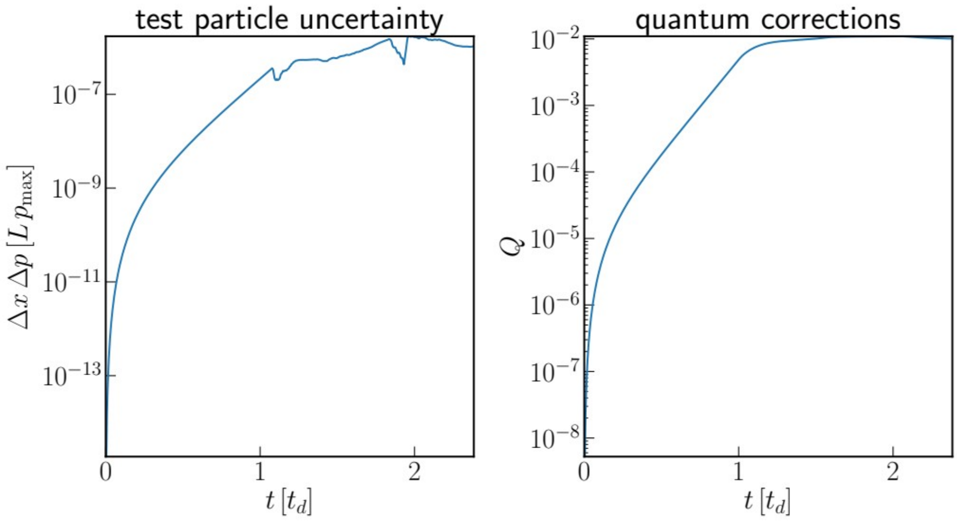

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- Small systems will be most effected by corrections but have the longest dynamical times
- Decoherence means states with large corrections are unlikely
- **Strong support that the predictions of the classical theory are accurate**

Questions

Extra slides: Truncated Wigner Approximation

Equations of motion for the truncated Wigner expansion [see Polkovnikov (Annals of Physics 2010)]

If I approximate my Wigner function as an ensemble of streams:

$$f_S[\psi(x),\psi^*(x)] = \frac{1}{N_s}\sum_{i}^{N_s} c_i\,\delta[\psi(x)-\psi_i(x,t)]\,\delta[\psi^*(x)-\psi_i^*(x,t)]$$

Is the independent classical evolution:

$$\begin{aligned}\partial_t\psi_i(x,t) &= -\frac{i}{\hbar}\left\{\,H_W[\psi_i(x),\psi_i^*(x)]\,,\,\psi_i(x,t)\,\right\}_c\\ &= -\frac{i}{\hbar}\frac{\partial H_W[\psi_i(x),\psi_i^*(x)]}{\partial\psi_i^*(x)}\end{aligned}$$

The same as approximating the evolution of the Wigner function to this order?:

$$\partial_t f_S[\psi(x),\psi^*(x)] \approx -\frac{i}{\hbar}\left\{\,H_W[\psi(x),\psi^*(x)]\,,\,f_S[\psi(x),\psi^*(x)]\,\right\}_c$$

Extra slides: Truncated Wigner Approximation

Yes

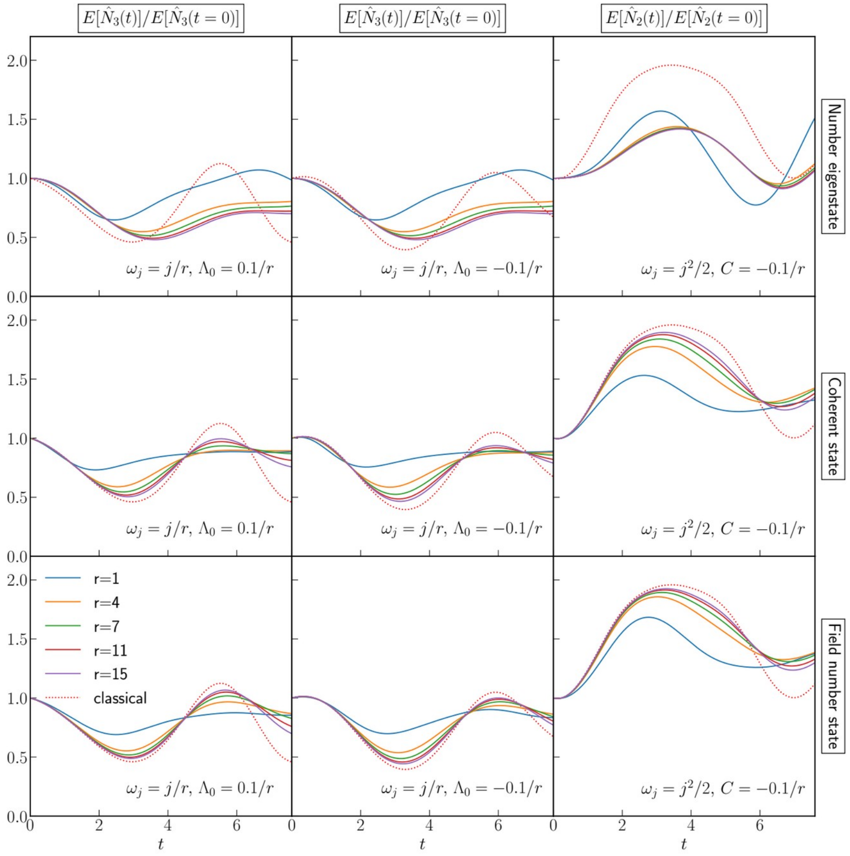
$$\delta[\Psi_i] \equiv \delta[\psi(x) - \psi_i(x,t)]$$

$$\partial_t \psi_i(x,t) = -\frac{i}{\hbar}\frac{\partial H_W[\psi_i(x), \psi_i^*(x)]}{\partial \psi_i^*(x)}$$

$$\begin{aligned}
\partial_t f_S[\psi(x), \psi^*(x)] &= \frac{1}{N_s}\partial_t \sum_i c_i\, \delta[\Psi_i]\, \delta[\Psi_i^*] \\
&= \frac{1}{N_s}\sum_i c_i\, (\partial_t \delta[\Psi_i])\, \delta[\Psi_i^*] + \delta[\Psi_i]\, (\partial_t \delta[\Psi_i^*]) \\
&= \frac{1}{N_s}\sum_i c_i \left(\frac{\partial \delta[\Psi_i]}{\partial \psi}\frac{\partial \psi_i(x,t)}{\partial t}\right) \delta[\Psi_i^*] + c.c. \\
&= \frac{1}{N_s}\sum_i c_i \left(\frac{\partial \delta[\Psi_i]}{\partial \psi}\frac{\partial \psi(x,t)}{\partial t}\right) \delta[\Psi_i^*] + c.c. \\
&= -\frac{i}{\hbar}\frac{1}{N_s}\sum_i c_i \left(\frac{\partial \delta[\Psi_i]}{\partial \psi}\frac{\partial H_W[\psi(x), \psi^*(x)]}{\partial \psi^*}\right) \delta[\Psi_i^*] - c.c. \\
&= -\frac{i}{\hbar}\frac{\partial H_W[\psi(x), \psi^*(x)]}{\partial \psi^*}\frac{\partial}{\partial \psi}\frac{1}{N_s}\sum_i c_i\, \delta[\Psi_i]\, \delta[\Psi_i^*] - c.c. \\
&= -\frac{i}{\hbar}\frac{\partial H_W[\psi(x), \psi^*(x)]}{\partial \psi^*}\frac{\partial f_S[\psi(x), \psi^*(x)]}{\partial \psi} - c.c. \\
&= -\frac{i}{\hbar}\left\{ H_W[\psi(x), \psi^*(x)]\, ,\, f_S[\psi(x), \psi^*(x)] \right\}_c .
\end{aligned}$$

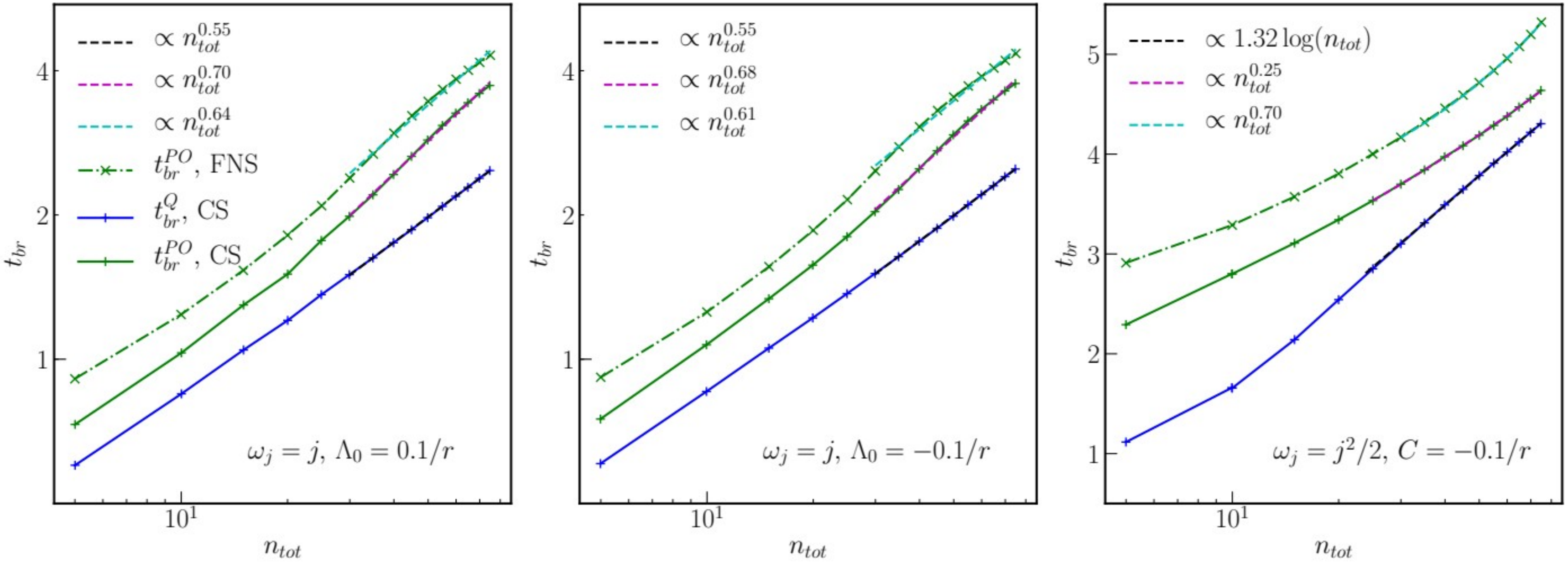
Extra slides: Other interactions/quantum states

- Looked at field number and number eigenstates
  - Coherent states are the states associated with the misalignment mechanism
  - Number state is immediately nonclassical
  - Field number state would be interesting follow up



Extra slides: Other interactions/quantum states

- Looked at contact interaction
  - Found spreads wavefunction too slowly (powerlaw, cf. Kerr oscillator)



Extra slides: Other interactions/quantum states

$$|\vec{z}\rangle_C = \bigotimes_{i=1}^{M} \exp\left[-\frac{|z_i|^2}{2}\right] \sum_{n_i=0}^{\infty} \frac{z_i^{n_i}}{\sqrt{n_i!}} |n_i\rangle$$

Coherent state

$$|\vec{z}\rangle_f = \sum_{\{n\}} \sqrt{n_{tot}!} \bigotimes_{i=1}^{M} \frac{z_i^{n_i}}{\sqrt{n_i!}} |n_i\rangle$$

Field number state