

# Can noncanonical domain walls play the role of halo dark matter on the galactic scale?

Fargiza A. M. Mulki  Hesti Wulandari Taufiq Hidayat

 fargiza@office.itb.ac.id

## 1 Introduction

Cosmological observations show that around 96% of the universe's energy budget is in the form of dark matter and dark energy, which are still poorly understood. The nature of the two entities is different either at small or large scales. Notwithstanding that the empirical properties of the two are very contradictive, several authors have attempted to unify dark matter and dark energy and perceive them as the same entity. Mulki et al. (2023a) show that the noncanonical domain walls (NDWs) with Lagrangian

$$\mathcal{L} = \left( -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right)^2 - V(\phi) \quad (1)$$

can serve as dark energy and, in others, as cosmological dark matter with the equation of state (EoS) parameter

$$w_{dw} = -\frac{3}{2} a^2. \quad (2)$$

In addition, Mulki et al. (2023b, *in preparation*) examined the interaction between domain walls with the Lagrangian (1). The authors found that two NDWs separated at a distance smaller than a value  $l_*$  will attract each other. They also showed that the  $l_*$  value is of the order of the cosmological scales.

Motivated by Mulki et al.'s results (2023a, 2023b), we can hypothesize that the network of NDWs may act as an attractive matter on the galactic scales. Therefore, in this work, we test this hypothesis and examine the behavior of NDWs at the galactic scales and their effect on the dynamics of a test particle.

At galactic scales, many observations confirm the requirement of dark matter to account for the flat rotation curve at the outer radii of galaxies. The notion of dark matter in galaxies has recently become extensive, especially since astronomers discovered the so-called core-cusp, missing satellite, and too-big-to-fail problems. These challenges enrich the issues and repertoire of dark matter in the galactic scales. Therefore, numerous models have been put forward to address these challenges, such as fuzzy, self-interacting, and solitonic dark matter (see *Particle Data Group 2023* for more details). Instead of examining these advanced problems, we address the noncanonical domain walls to the basic evidence of dark matter in the galactic scales, i.e., the flat rotation curve of galaxies.

## 2 Objective

To examine whether NDWs with the Lagrangian (1) can play the role of halo dark matter in the galactic scales, i.e., exhibiting a flat rotation curve at large radii.

## 3 Field equation

- Spacetime background:

$$ds^2 = - \left( 1 - \frac{R_s}{r} \right) dt^2 + \left( 1 - \frac{R_s}{r} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (3)$$

where  $R_s \equiv 2GM$

- Assumptions for the field  $\phi$  and its spacetime:
  - Weak gravitational field limit ( $R_s/r \ll 1$ ) (for the large radii cases)
  - Vacuum dominated phase (vacuum  $\gg \phi$ )  $\rightarrow V(\phi) = V \approx \text{constant}$  (for a consistency with its cosmological solution (Mulki et al. 2023b))
  - Perfect fluid for general field  $\phi$
- Equation of motion (EoM) of Lagrangian (1) in background (3):

$$\frac{d^2 \phi}{dr^2} + \frac{R_s}{r} \frac{d\phi}{dr} = 0 \quad (4)$$

- For an NDW with  $D$ -spatial dimensions:  
The energy density and pressure:

$$\rho_\phi = V + \frac{D^2}{4} (\partial_r \phi)^4 \quad (5)$$

$$P_\phi = V + D(\partial_r \phi)^4 - \frac{D^2}{4} (\partial_r \phi)^4 \quad (6)$$

## 4 Polytropic NDWs

- We assume that the EoS of NDWs at the galactic scales obeys the polytropic relation

$$P = P_* \left( \frac{\rho}{\rho_*} \right)^\gamma \quad (7)$$

- Using Eq. (5), (6) and (4), it follows that  $\gamma = 1$  is a polytropic index solution for NDWs, regardless of its spatial dimension  $D$ .

## 5 Dynamics of a test particle

- The pressure gradient inside an NDWs halo is given by the TOV equation

$$\frac{dP}{dr} = -\frac{1}{r} (\rho + P) \left( \frac{GM}{r} + 4\pi G r^2 P \right) \left( 1 - \frac{2GM}{r} \right)^{-1} \quad (8)$$

- The EoS of NDWs is described by Eq. (7) with  $\gamma = 1$
- Approximate conditions:

- Weak gravitational field limit ( $2GM/r \ll 1$ )
- Pressure dominated dynamics ( $4\pi G r^2 P \gg GM/r$ )

- Solution:

$$P(r) \approx \left( \frac{2\pi G \rho_*}{P_*} (r^2 - r_*^2) + \frac{1}{P_*} \right)^{-1} \quad (9)$$

- Rotational velocity of a test particle in a distance  $r$ :

$$v^2 \approx \frac{8\pi G P_*}{\rho_*} \frac{r^2}{(r^2 - r_*^2)} \quad (10)$$

- For large  $r$

$$v^2 \approx \frac{8\pi G P_*}{\rho_*} = \text{constant} \quad (11)$$

## 6 Discussion

The NDWs embedded in the spherical symmetry background have a polytropic index  $\gamma = 1$  so that the equation of state takes the simple form, i.e.,  $P(r) \propto \rho(r)$ . The extensive presence of the NDWs in a galaxy can provide an excess attractive pressure, forcing a test particle at large radii to move faster than that of the Keplerian case. Thus, the NDWs can play the role of halo dark matter that explains the flat rotation curve at the outer radii of galaxies.

## References

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